Quick multiplication.

OF

- multiply by 5 — $\times 10 \div 2$
  - 25 — $\times 100 \div 4$
  - 45 — $\times 90 \div 2$
  - 75 — $\times 300 \div 4$
  - 225 — $\times 900 \div 4$
  - 275 — $\times 1100 \div 4$
  - 125 — $\times 500 \div 4$

- multiply b 11, 22 = (2×11), 33 = (3×11) etc.
  eg $777 \times 11 = 8547$ ($777 \cdot 11 = 8\ 54\ 7$) fixed

- multiplyg nos w̄ same 1st figures & sum of 2nd figures = 10
  eg $42 \times 48 = (4 \times 5) | (2 \times 8) = 2016$ (5 fr. 1 no. up)

- multiplyg nos. w̄ 1st figures = 10; last figures are same
  eg. $44 \times 64 = (4 \times 6 + 4) | (4 \times 4) = 2816$
  (add 0 B4 unit digit if multiplicatn of last figures < 10)

- multiplyg nos. w̄ 1st nos. = 10, 2nd nos. are same
  eg $46 \times 55 = (4 \times 5 + 5) | (6 \times 5) = 2530$
  (add 0 B4 unit digit if multiplicatn < 10)

- multiplyg nos. just over 100
  eg. $108 \times 109 = (108 + 9) | (9 \times 8) = 11772$

- $(x+y)^8 = {}^8C_8 x^8 + {}^8C_7 x^7 y^1 + {}^8C_6 x^6 y^2 + {}^8C_5 x^5 y^3 + \ldots {}^8C_1 x y^7 + {}^8C_0 y^8$

Number Properties.

OF

Prime Nos

- All prime nos > 2 are odd.
- Defn: +ve integer that has 2 diff +ve divisors: 1 & itself.
- 0 is an even integer & prime no.; 1 is not a prime no.
- If N is a no. such that $N = (a^m)(b^n)(c^p)$ ... where a, b, c are prime nos

$\therefore$ no. of divisors of N inc N $= (m+1)(n+1)(p+1)\ldots$

sum of " " " $= \dfrac{(a^{m+1}-1)}{(a-1)} \times \dfrac{(b^{n+1}-1)}{(b-1)} \times \dfrac{(c^{p+1}-1)}{(c-1)} \ldots$

Percentages

- No. $\uparrow x\%$ then $\downarrow x\%$ $\longrightarrow$ Net $\Delta = \downarrow \dfrac{x^2}{100}\%$

No. $\uparrow x\%$ then $\downarrow y\%$ $\longrightarrow$ Net $\Delta = \uparrow \left(x - y - \dfrac{xy}{100}\right)\%$ if +ve

$\downarrow$ " if −ve

If order of $\uparrow/\downarrow$ is Δed, Net $\Delta = 0$ (result unaffected)

No. $\uparrow x\%$ then $\uparrow y\%$, Net $\Delta = \uparrow \left(x + y + \dfrac{xy}{100}\right)\%$

Number Properties.

eg 12, 180, 50.
12 : 2 . 2 . 3
180 : 2 . 5 . 2 . 2 . 2
50 : 2 . 5 . 5

HCF = 2
LCM $= 2^4 \cdot 5^2 \cdot 3$

LCM / HCF

(a) Find all prime factors for all nos. in question.

LCM = smallest +ve integer that is multiple of all nos = highest power of common factors (x)

HCF = largest +ve integer that ÷ all nos = least power of common factors (x).

- Total no. of prime factors of a no. ⇒ refer to prime nos. sheet.
- LCM of a fraction = LCM of numerator / HCF of denominator

HCF of a fraction = HCF of numerator / LCM of denominator

Fractions

- when fractions ÷ 0 & 1 are squared, they get smaller.
- if $x > 1$, $x^2 > x$; if $0 < x < 1$, $x^2 < x$
- when fractions ÷ 0 & 1 are rooted, they get bigger.

- $\frac{a}{b^2} + \frac{b}{a^2} > \frac{1}{a} + \frac{1}{b}$ if $a > 0$, $b > 0$, $a \neq b$
- If $a \neq b$ are real nos, $a^2 + b^2 > 2ab$
- If $a \neq b \neq c$ are real nos, $a^2 + b^2 + c^2 > ab + bc + ca$
- If $a > 0$, $b > 0$, $a \neq b$ then $\frac{a}{b} + \frac{b}{a} > 2$

Co-od Geom

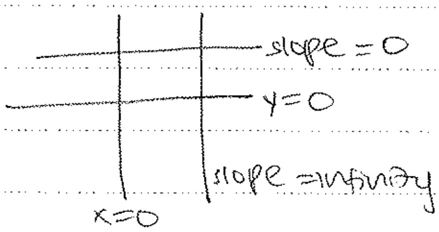
OF

$P_1(X_1, Y_1)$; $P_2(X_2, Y_2)$; $P_3(X_3, Y_3)$

- Dist $\%$. P_1 & P_2 = $\sqrt{(X_1-X_2)^2 + (Y_1-Y_2)^2}$
- co-ods of pt. dividing line segment P_1P_2 in ratio r/s is
  $$\frac{rX_2 + sX_1}{r+s} \;,\; \frac{rY_2 + sY_1}{r+s}$$
- when $r=s$, midpt of line P_1P_2 has co-ods: $\frac{(X_2+X_1)}{2}$, $\frac{(Y_2+Y_1)}{2}$
- slope m of line P_1P_2 = $(Y_2-Y_1)/(X_2-X_1)$

- 2 non-vertical lines are parallel if their slopes are parallel .ie $m_1 = m_2$
  " " " " " perpendicular " " " have product of $-1 = m_1 \cdot m_2$

- A line is a bisector of another line if it cuts it at 90° & into 2 equal lengths
- Line $Y=X$ acts as a mirror. images go (X,Y) & (Y,X) & line is perpendicular.
- Area of $\triangle P_1P_2P_3 = \left|\frac{1}{2}\left[X_1(Y_2-Y_3) + X_2(Y_3-Y_1) + X_3(Y_1-Y_2)\right]\right|$
- Dist of $P_1(X_1,Y_1)$ from line $ax+by+C=0$ is given by:
  $$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$

- 

RATIOS / INTEREST RATES / AVERAGES

OF

Ratios

- Ratio of X to Y = $\frac{X}{Y}$, when calculating, use form X/Y to work out ans
- 2 nos. are reciprocals iff their product = 1
- B produces 10% more than A ie $B = \frac{110}{100}A$ NOT $\frac{90}{100}B = A$.
- inverse proportionality ie $X \propto \frac{1}{y}$

Interest Rates

- Final bal = principle × $\left(1+\frac{i/r}{c}\right)^{(\text{time} \times c)}$

  c: no. of times compounded annually.

  i/r: in ē form of decimals eg. 0.8 (ie 80%)

  time: in ē form of years
- Note: usually calculate simple interest + something bigger.
- When i/r for successive fixed periods are $r_1\%$, $r_2\%$, $r_3\%$ … final amt A ⇒

  $A = P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\ldots$

Averages

Note: median if nos are consecutive, median = $\frac{a+b}{2}$
a: 1st term
b: last term.

- mean: sum of observations / no. of observations.
- mode: value of term $\bar{w}$ greatest frequency.
- median: (a) arrange nos in ascend^g / descend^g order

  (b) find no. of terms.

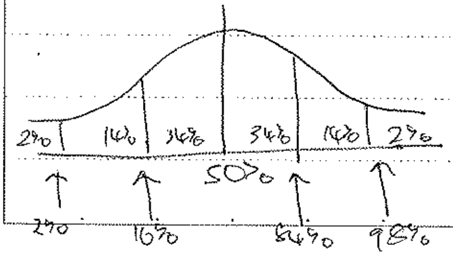
  – if even, median = $\frac{1}{2}\left[\frac{n}{2} + \frac{(n+1)}{2}\right]$

  – if odd, median = $\frac{(n+1)}{2}$
- std dev (a) find ave of each set

  (b) find X = each value – ave value for each vall

  (c) $Y = X^2$ for each value

  (d) find ave ~~of~~ all Y's

  (e) std dev = $+\sqrt{Y_{ave}}$
- Range = minimum – maximum.
- 

  34 : 14 : 2 ie 1 : 2 : 3 sd fr. mean.

Rates

OF

Distance / speed / Time

- same direction : key pt when 1 overtakes other : same dist travelled
- opp. direction : total dist travelled = sum of indv dist travelled.

(both cases apply to overtaking)

- round trip : dist to & fro are equal.
- organise data in boxes by D, S, T ū rows represent^g mov^g object

	S	· T	= D
G	12	x	$12x$
M	36	$x - 1/3$	$36(x - 1/3)$

$\therefore 12x = 36(x - 1/3)$

eg. G travels at 12mph. M leaves 20 min later & travels at 36mph.
At what dist would M overtake G.

- Time taken by train of l m to pass pole = time taken to travel l m.

Time taken by train of l m to pass object b m = time taken to travel $(l+b)$ m

- 2 bodies mov^g in same direct^n at u m/s & v m/s ; relative speed = $(u-v)$ m/s

" " " " opp " " " " " " ; " " $= (u+v)$ m/s

- 2 bodies, A & B, lengths a m & b m, speed u m/s, v m/s

- opp direct^n : time taken to X each other = $(a+b)/(u+v)$ sec.
- same direct^n : time taken for overtaking = $(a+b)(u-v)$ sec.

- 2 bodies start at pts A & B at same time, mov^g towards each other. After cross^g take a sec & b sec to reach pts B & A.

A's speed : B's speed $= \sqrt{B} : \sqrt{A}$

Rates

OF

Work & Time

- Pipe A can fill a tank in x hrs & Pipe B empty it in y hrs. If both pipes are open, tank will be filled in $\frac{(xy)}{(y-x)}$ hrs
- Pipe A can fill a cistern in x hrs but due to leak, does it in y hrs.
  $\therefore$ Time taken by leak to empty cistern = $\frac{xy}{(y-x)}$ hrs.

Dilution / mixture

- $(\%_1)(amt_1) + (\%_2)(amt_2) = \%_T(amt_1 + amt_2)$
-

	amt	x	%	=	total
1	50		10		500
2	x		30		$30x$
mixture.	$x+50$		20		$20(x+50)$

$\therefore \; 500 + 30x = 20(x+50)$

- (Diff % weaker & desired)(amt_W) = (diff % stronger & desired)(amt_S)

- A : quantity N_1 ; strength V_1
  B : " N_2 ; " V_2
  mean strength $V_m = (V_1N_1 + V_2N_2) / (N_1+N_2)$
  $\therefore \frac{N_1}{N_2} = (V_2 - V_m) / (V_m - V_1)$
  $\therefore N_1 : N_2 : N_3 = (V_2-V_m)(V_3-V_m) : (V_m-V_1)(V_3-V_m) : (V_2-V_m)(V_m-V_1)$

- Vessel contains M units of A & B. X units are taken out & replaced by X units of B only. This is done n times
  $\therefore$ Amt of A left / Amt of original A $= \left(1 - \frac{X}{M}\right)^n$

- Vessel contains M units of A only. X units are taken out & replaced by X units of B only. This is done n times
  $\therefore$ Amt of A left $= M\left[1 - \frac{X}{M}\right]^n$

$\sqrt{2} = 1.4$

$\sqrt{3} = 1.7$

Squares and Cubes

$\pi = \frac{22}{7} = \frac{2 \times 11}{7}$

$\sqrt{2} = 1.4$

$\sqrt{3} = 1.7$

Number (x)	Square (x^2)	Cube (x^3)
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	-
8	64	-
9	81	-
10	100	-
11	121	-
12	144	-
13	169	-
14	196	-
15	225	-
16	256	-
17	289	
18	324	
19	361	
21	441	
22	484	
23	529	
24	576	
25	625	
30	900	

Fractions and Percentage:

Fraction	Decimal	Percentage
1 / 2	0.5	50
1 / 3	0.33	33 1/3
2 / 3	0.66	66 2/3
1 / 4	0.25	25
3 / 4	0.75	75
1 / 5	0.2	20
2 / 5	0.4	40
3 / 5	0.6	60
4 / 5	0.8	80
1 / 6	0.166	16 2/3
5 / 6	0.833	83 2 / 3
1 / 8	0.125	12 1 / 2
3 / 8	0.375	37 1 / 2
5 / 8	0.625	62 1 / 2
7/8	0.875	87 1/2
1/9	0.111	11

OF

Absolute Inequality Eqs.

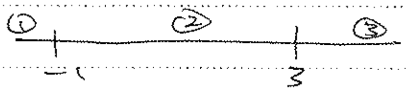
eg $|x-3| \Rightarrow |x-3| = x-3$ when $x > 3$

$= -(x-3)$ when $x < 3$

can also think of it as distance of x from 3.

eg $|x+1| + |x-3| = 6$ — 4 possible scenarios. look at critical pts.

critical pts are -1 & 3



$\therefore$ 3 scenarios.

when $x < -1$, $(x+1)$ & $(x-3)$ is $-ve$ $\therefore -(x+1)-(x-3)=6$ ie $x=-2$

check that $x=-2$ is <-1

when $-1 < x < 3$, $(x+1)$ is $+ve$, $(x-3)$ is $-ve$

$\therefore (x+1)-(x-3) = 6$ ie $4=6$ (X) no soln

when $x > 3$, $(x+1)$ is $+ve$, $(x-3)$ $+ve$ $\therefore (x+1)+(x-3)=6$

$\therefore x = 4$ check that $x=4$ is >3

Rules

1) cannot multiply / divide by a variable unless you know ē sign.

2) cannot square root / square both sides ie $x > y \neq x^2 > y^2$

3) $|x+y| \leq |x| + |y|$

Trick Qns

OF

- calculating value of expression "5X+6Y" vs value of unknowns X & Y in 5X + 6Y.
- Backsolving: start w (E) & work to (A) for choices w variables
For ans. w more concrete values, start with (C) then decide if ans. shld be bigger ie (D) or (E), or smaller ie (A) or (B).
- Picking nos: try all possible choices: +ve, −ve, 0, fractions.
  −2, −1, −0.5, 0, 0.5, 1, 2.
- purchase price ≠ mkt value
- 12mn to 12 noon does not mention days in between − assumptn!
- $x^2 = 9y^2 \neq x^2 > y^2$ note what if $x = y = 0$.

Important math eqs.

OF

1) Highest power of 6 that divides 20! completely.

A) Find prime factors of 6 ie 2×3.

B) consider largest prime factor ie 3

C) $\frac{20}{3} = 6$, $\frac{20}{9} = 2$

$\therefore 6^8$ completely divides 20

OR multiples of 3 in 20 : 3, 6, 9, 12, 15, 18

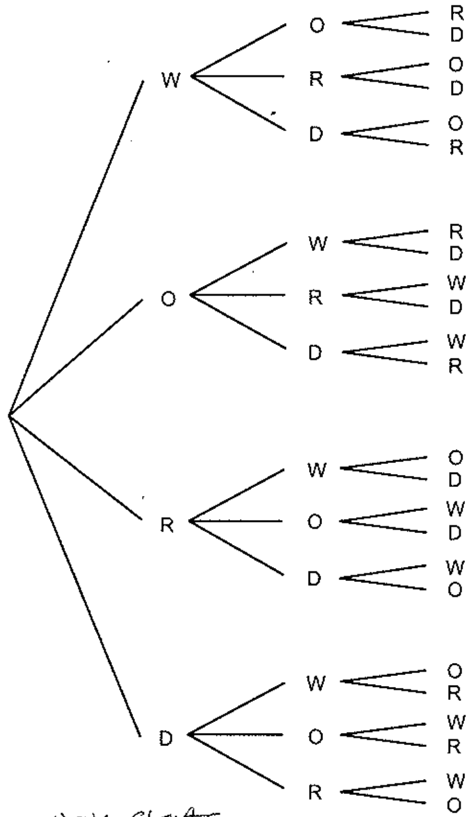
$= 3^1, 3^1 \times 2, 3^2, 3 \times 4, 3 \times 5, 3^2 \times 2$.

count the no. of 3's = 8

$\therefore 6^8$ completely divides 20.

COUNTING, PERMUTATIONS AND COMBINATIONS

MULTIPLICATION PRINCIPLE



Consider the 3 letter words that can be made from the letters WORD if no letter is repeated. These can be listed by means of a tree diagram.

There are:

- 4 ways of choosing the 1st letter
- 3 ways of choosing the 2nd letter
- 2 ways of choosing the 3rd letter
- number of words = 4 x 3 x 2 = 24

This is an illustration of the *multiplication principle* ie. if several operations are carried out in a certain order, then the number of ways of performing all the operations is the product of the numbers of ways of performing each operation. ie. no. of ways to order a set of objects.

The principle is equivalent to filling in pigeonholes:

1st letter		2nd letter		3rd letter			
$\square$		$\square$		$\square$			
4	x	3	x	2	=	24	ie. 4!

Note: 1st operation should finish B4 2nd operation starts.

ADDITION PRINCIPLE

Consider the 3 letter words starting <u>or</u> finishing with O that can be made from the letters WORD if no letter is repeated. Now words starting or finishing with O are *mutually exclusive* ie. they do not overlap. Therefore we can find the number starting with O and the number finishing with O and add the two numbers.

<u>starting with O</u>

1st letter		2nd letter		3rd letter		
$\square$		$\square$		$\square$		
1	x	3	x	2	=	6

<u>finishing with O</u>

1st letter		2nd letter		3rd letter		
$\square$		$\square$		$\square$		
3	x	2	x	1	=	6

number of words starting or finishing with O = 6 + 6 = 12

This is an illustration of the *addition principle* ie. if two operations are mutually exclusive (ie. they do not overlap), then the number of ways of performing one operation <u>or</u> the other is the sum of the numbers of ways of performing each operation.

FACTORIAL NOTATION

In applying the multiplication principle, factorial notation can be useful eg. the number of 6 letter words that can be made from the letters FACTOR is 6! = 6 x 5 x 4 x 3 x 2 x 1.

Note: $0! = 1! = 1$

$4! = 4 \times 3 \times 2 \times 1$

"TOGETHER" ARRANGEMENTS

In this type of problem, we need to count arrangements where some of the objects must remain together. The multiplication principle applies and we use a "treat as one" technique.

Eg. 3 science, 4 mathematics and 5 history books are arranged on a shelf. How many arrangements are possible if the books from each subject are to be together?

treat the books for each subject as one book:
number of arrangements = 3!

Note: 1st operat^n shld finish B4 2nd one starts etc.

number of ways of arranging the science books = 3!
number of ways of arranging the mathematics books = 4!
number of ways of arranging the history books = 5!

total number of arrangements = 3! x 3! x 4! x 5! = 103680

ARRANGEMENTS INVOLVING IDENTICAL OBJECTS

Consider the number of arrangements of the letters EMPLOYEE. If all 8 letters were different, then the number of arrangements would be 8! but this number involves counting arrangements more than once. Eg. The 8! arrangements includes 6 versions of EEEMPLOY:

ie. $\dfrac{(\text{no. of items in set})!}{(\text{indepevent})!\ (\text{indep. event})!}$

$E_1E_2E_3$MPLOY
$E_1E_3E_2$MPLOY
$E_2E_1E_3$MPLOY
$E_2E_3E_1$MPLOY
$E_3E_1E_2$MPLOY
$E_3E_2E_1$MPLOY

Similarly every arrangement occurs 6 times in the total of 8! (6 is the number of arrangements of the 3 E's ie. 3!).

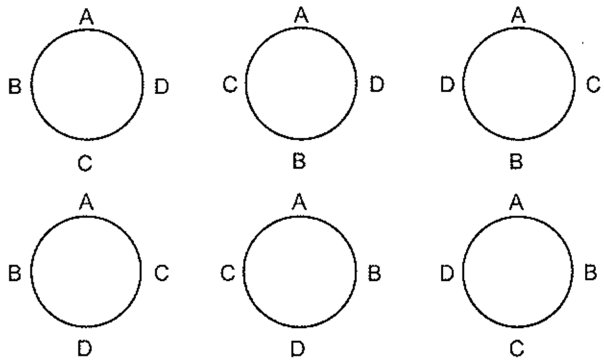
$$\text{number of distinct arrangements} = \frac{8!}{3!} = 6720$$

This idea can be extended to problems where more than one type of object is repeated:

$$\text{number of distinct arrangements of the letters MISSISSIPPI} = \frac{11!}{4!4!2!} = 34650$$

CIRCULAR ARRANGEMENTS

Consider the different arrangements when 4 people sit in a circle. There are 3! arrangements (not 4! as might be expected) as it does not matter where the first object is placed.



In general, n objects can be arranged in a circle in $(n-1)!$ ways. (when clockwise & anticlockwise arrangements are diff)

if same = $\dfrac{(n-1)!}{2}$

PERMUTATIONS AND COMBINATIONS

Choosing objects from a collection of different objects can be done in several ways. Two particular ways are given the names permutation and combination.

Choosing r objects without repetition from n different objects such that order matters is called a *permutation* and the number of such permutations is denoted by ${}^nP_r$.

Choosing r objects without repetition from n different objects such that order does not matter is called a *combination* and the number of such combinations is denoted by ${}^nC_r$.

Example of permutations:
How many ways can a committee of 3 be selected from 7 people A,B,C,D,E,F,G so that there is a president, a vice-president and a secretary? Using the multiplication principle:

pres.		vice-pres.		sec.		
$\square$		$\square$		$\square$		
7	x	6	x	5	=	210

$$\therefore \quad {}^7P_3 = 7\times 6\times 5 = \frac{7!}{4!} \quad \text{and in general} \quad {}^nP_r = \frac{n!}{(n-r)!}$$

NB. Permutation problems are usually best done using the multiplication principle rather than permutation notation.

Example of combinations:
How many ways can a committee of 3 be selected from 7 people A,B,C,D,E,F,G so that each member of the committee is equal? The number of permutations 210 is too large because every combination occurs 6 times in the 210 permutations. Eg. The following 6 permutations each give rise to the same combination (6 is the number of arrangements of 3 objects ie. 3!):

ABC ACB BAC BCA CAB CBA

$$\therefore \quad {}^7C_3 = \frac{7\times 6\times 5}{3!} = \frac{7!}{3!4!} \quad \text{and in general} \quad {}^nC_r = \frac{n!}{r!(n-r)!}$$

SPECIAL COMBINATIONS

Choosing 7 from 7 so that order does not matter can only be done 1 way:

$$\therefore \quad {}^7C_7 = 1 \quad \text{and in general} \quad {}^nC_n = 1$$

Choosing 0 from 7 so that order does not matter can only be done 1 way:

$$\therefore \quad {}^7C_0 = 1 \quad \text{and in general} \quad {}^nC_0 = 1$$

This leads to the conclusion that 0! must be given the value 1 because ${}^7C_7 = \frac{7!}{7!0!} = 1$.

sum of combination of n distinct things $= 2^n$
eg. Door can be opened w̄ code of 5 buttons 1, 2, 3, 4, 5. code can be pressg 1 button, or 2, 3, 4, 5. How many codes are there?
$= 2^5 - 1$ (1 because not pressg any, ${}^5C_0 = 1$)

COMBINATIONS - INCLUSIONS / EXCLUSIONS

Consider the number of ways a committee of 3 can be selected from 7 people A,B,C,D,E,F,G (order does not matter) if:

B must be included (select 2 from A,C,D,E,F,G) $^6C_2$

D must be excluded (select 3 from A,B,C,E,F,G) $^6C_3$

C and E cannot be chosen together $^5C_3+^5C_2+^5C_2$

What is the justification for the last answer?

CHOOSING SETS OF OBJECTS WITH DISTINCT SUBSETS

In these problems, find the number of ways of choosing each subset and then use the multiplication principle. Eg. 6 people are chosen (order does not matter) from 5 Queenslanders, 4 Tasmanians and 3 Victorians:

2 from each state are chosen:

number of ways of choosing 2 Queenslanders = $^5C_2$

number of ways of choosing 2 Tasmanians = $^4C_2$

number of ways of choosing 2 Victorians = $^3C_2$

total number of ways = $^5C_2 \times ^4C_2 \times ^3C_2$

at least 3 Queenslanders are chosen:

number of ways of choosing 3 Queenslanders and 3 others = $^5C_3 \times ^7C_3$

number of ways of choosing 4 Queenslanders and 2 others = $^5C_4 \times ^7C_2$

number of ways of choosing 5 Queenslanders and 1 others = $^5C_5 \times ^7C_1$

total number of ways = $^5C_3 \times ^7C_3 + ^5C_4 \times ^7C_2 + ^5C_5 \times ^7C_1$

at least one Queenslander is chosen:

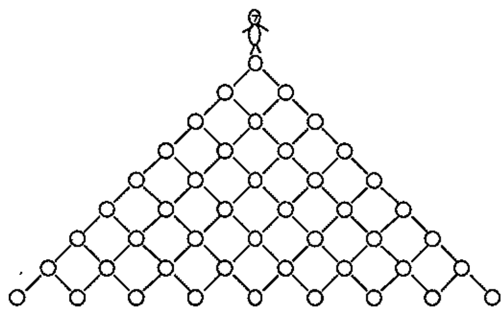
number of ways of choosing without restrictions = $^{12}C_6$

number of ways of choosing with no Queenslanders = $^7C_6$

total number of ways = $^{12}C_6 - ^7C_6$

PASCAL'S TRIANGLE

Suppose that you start from the top of the triangular arrangement of spaces shown on the attached sheet.



In each space, write the number of shortest possible routes to that space. These numbers form *Pascal's triangle*.

Can you see the pattern which takes you from one row to the next? Justify this pattern in terms of the shortest route problem.

The shortest route problem can be used to explain why each row can be written as combinations eg. the last row is:

$$^8C_8 \quad ^8C_7 \quad ^8C_6 \quad ^8C_5 \quad ^8C_4 \quad ^8C_3 \quad ^8C_2 \quad ^8C_1 \quad ^8C_0$$

Each number in Pascal's triangle (except those on the outside) can be found by adding the pair of numbers immediately above. This recurrence relationship can be written as:

$$^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

As well as justifying this recurrence relationship in terms of the shortest route problem, it can be derived in other ways:

- algebraically by writing $^nC_r = \dfrac{n!}{r!(n-r)!}$ etc.
- by considering the selection of r objects from two collections - collection A containing n objects and collection B containing one object.

Consider sloping rows of Pascal's triangle. In turn, they give:

- ones $\{1,1,1,1,...\}$
- natural numbers $\{1,2,3,4,5,\ldots\}$
- triangle numbers $\{1,3,6,10,15,\ldots\}$
- tetrahedral numbers $\{1,4,10,20,35,\ldots\}$

What is the connection between natural numbers, triangle numbers and tetrahedral numbers? How do triangle numbers and tetrahedral numbers get their names? What is the n th triangle number and the n th tetrahedral number as a combination? Write these combinations as algebraic expressions in n.

BINOMIAL THEOREM

Consider the *binomial expansion*:

$$(x+y)^8 = (x+y)(x+y)(x+y)(x+y)(x+y)(x+y)(x+y)(x+y)$$

The number of x^5y^3 in the expansion is the same as the number of ways of selecting 5 x's from the 8 available ie. $^8C_5$. This leads to the *binomial theorem* for binomial expansions $(x+y)^n$ where n is a positive integer. The theorem gives the coefficients as combinations. Eg. for $n = 8$:

$$\begin{aligned}&(x+y)^8\\&={}^8C_8\,x^8+{}^8C_7\,x^7y+{}^8C_6\,x^6y^2+{}^8C_5\,x^5y^3+{}^8C_4\,x^4y^4+{}^8C_3\,x^3y^5+{}^8C_2\,x^2y^6+{}^8C_1\,x\,y^7+{}^8C_0\,y^8\\&=x^8+8x^7y+28x^6y^2+56x^5y^3+70x^4y^4+56x^3y^5+28x^2y^6+8x\,y^7+y^8\end{aligned}$$

From the earlier work on Pascal's triangle, we can also say that the coefficients of a binomial expansion are given by the appropriate row of Pascal's triangle. Substituting $x = y = 1$ in the binomial theorem shows that the sums of the rows of Pascal's triangle are powers of 2.

$$\text{Prob} = \frac{\#\text{ favourable outcomes}}{\#\text{ possible outcomes}}$$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

ME events : $P(A \cap B) = 0$

$\therefore P(A \cup B) = P(A) + P(B)$

Inde. events $P(A \cap B) = P(A)P(B)$

$\therefore P(A \cup B) = P(A) + P(B) - P(A)P(B)$

Probability can be studied in conjunction with set the
particularly useful in analysis.
The probability of a certain event occurring, for exam
The probability of a different event occurring can be
two events A and B,
$P(A) + P(B) - P(A \cap B) = P(A \cup B)$
$P(A \cap B)$ represents the probability of A AND B occu
probability of A OR B occurring.

multiplicat'y rule.

Mutual Exclusive Events – not independent

Events A and B are mutually exclusive if they have no events in common. In other words, if A occurs B cannot occur and vice-versa. On a Venn Diagram, this would mean that the circles representing events A and B would not overlap.

If, for example, we are asked to pick a card from a pack of 52, the probability that the card is red is ½ . The probability that the card is a club is ¼. However, if the card is red it can't be a club. These events are therefore mutually exclusive.
If two events are mutually exclusive, $P(A \cap B) = 0$, so
$P(A) + P(B) = P(A \cup B)$: Addition Rule

Independent Events – not mutually exclusive

Two events are independent if the first one does not influence the second. For example, if a bag contains 2 blue balls and 2 red balls and two balls are selected randomly, the events are:
a) independent if the first ball is replaced after being selected
b) not independent if the first ball is removed without being replaced. In this instance, there are only three balls remaining in the bag so the probabilities of selecting the various colours have changed.
Two events are independent if (and only if):
$P(A \cap B) = P(A)P(B)$
This is known as the multiplication law.
Addition Rule : $P(A \cup B) = P(A) + P(B) - P(A)P(B)$

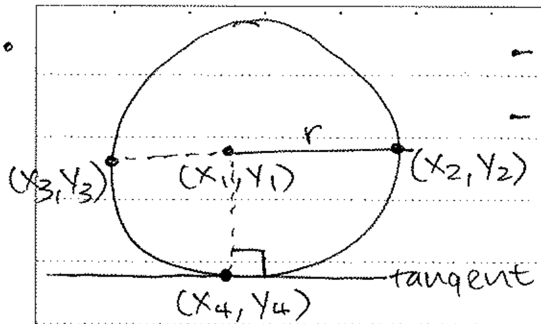
Conditional Probability

Conditional probability is the probability of an event occurring, given that another event has occurred. For example, the probability of John doing mathematics at A-Level, given that he is doing physics may be quite high. $P(A|B)$ means the probability of A occurring, given that B has occurred. For two events A and B,
$P(A \cap B) = P(A|B)P(B)$
and similarly $P(A \cap B) = P(B|A)P(A)$.
If two events are mutually exclusive, then $P(A|B) = 0$.

For independent events, $P(B|A) = P(B)$
$P(A|B) = P(A)$

Baye's Theorem $\Rightarrow$ $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(B|A)P(A)}{P(B)}$

Circles



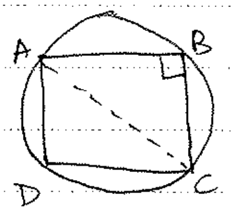
- Area $= \pi r^2$
- Circumference $= 2\pi r$

- Eq^{n} of circle: $(x-x_1)^2 + (y-y_1)^2 = r^2$ given (x_1, y_1) in centre & r
  : $(x-x_3)(x-x_2) + (y-y_3)(y-y_2) = 0$ given line $(x_2, y_2)-(x_3, y_3)$

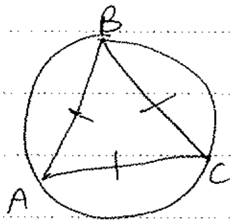
- Eqn of tangent: $x \cdot x_4 + y \cdot y_4 + r^2$ given circle $x^2+y^2=r^2$
- condition for $y=mx+c$ to be tangent to circle $x^2+y^2=r^2$ is: $c^2=r^2(1+m^2)$

- 2 circles will touch / intersect each other if distance b/ centres d is such tt
  $R-r \leq d \leq R+r$ where R & r are $\bar{e}$ 2 radii

- $\frac{\text{Area circle 1}}{\text{Area circle 2}} = \left(\frac{\text{Radius}_1}{\text{Radius}_2}\right)^2$
- Arc length $= \left(\frac{x}{360}\right) 2\pi r$
  Area of sector $= \left(\frac{x}{360}\right) \pi r^2$
- Equal chords are equidistant from centre



- circle's diameter = square's diagonal
- if $\triangle$ in ◯ such tt 1 side is diameter, $\triangle$ is ⊾



Radius of circumcircle of $\triangle = \frac{2}{3}$ (height of $\triangle$)
Arc AB = Arc BC = Arc AC

Math B - Lesson Page

Angles Formed by Radii, Chords, Tangents, Secants

Formulas for Working with Angles in Circles

(*Intercepted arcs* are arcs "cut off" or "lying between" the sides of the specified angles.)

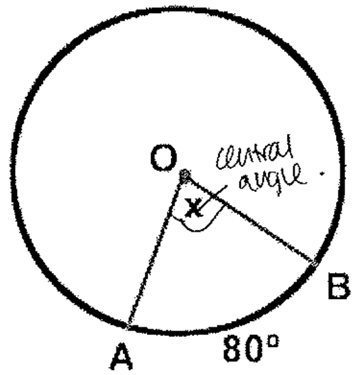
There are basically five circle formulas that you need to remember:

1. Central Angle:

A central angle is an angle formed by two intersecting radii such that its vertex is at the center of the circle.

$$\boxed{\text{Central Angle} = \text{Intercepted Arc}}$$

<AOB is a central angle.
Its *intercepted arc* is the minor arc from A to B.
m<AOB = 80°

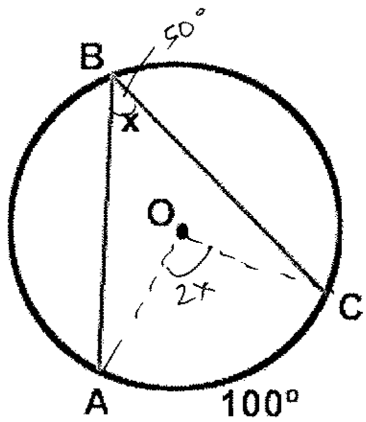


2. Inscribed Angle:

An inscribed angle is an angle with its vertex "on" the circle, formed by two intersecting chords.

$$\boxed{\text{Inscribed Angle} = \frac{1}{2}\,\text{Intercepted Arc}}$$

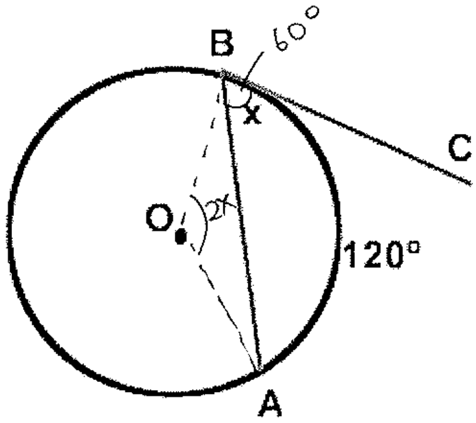
<ABC is an inscribed angle.
Its *intercepted arc* is the minor arc from A to C.
m<ABC = 50°



3. Tangent Chord Angle:

An angle formed by an intersecting tangent and chord has its vertex "on" the circle.

Tangent Chord Angle = $\frac{1}{2}$ Intercepted Arc



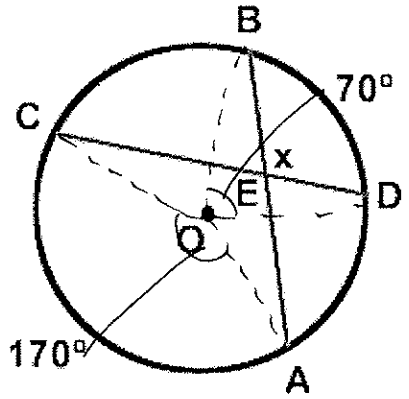
<ABC is an angle formed by a tangent and chord. Its *intercepted arc* is the minor arc from A to B. m<ABC = 60°

4. Angle Formed Inside of a Circle by Two Intersecting Chords:

When two chords intersect "inside" a circle, four angles are formed. At the point of intersection, two sets of vertical angles can be seen in the corners of the X that is formed on the picture. Remember: vertical angles are equal.

Angle Formed Inside by Two Chords = $\frac{1}{2}$ **Sum** of Intercepted Arcs

Once you have found ONE of these angles, you automatically know the sizes of the other three by using your knowledge of vertical angles (being equal) and adjacent angles forming a straight line (adding to 180).



<BED is formed by two intersecting chords.

Its *intercepted arcs* are $\overset{\frown}{BD}$ and $\overset{\frown}{CA}$. [Note: the intercepted arcs belong to the set of vertical angles.]

$$m\angle BED = \frac{1}{2}(70+170) = \frac{1}{2}(240) = 120°$$

also, m<CEA = 120° (vetical angle)
m<BEC and m<DEA = 60° by straight line.

5. Angle Formed Outside of a Circle by the Intersection of:

"Two Tangents" or "Two Secants" or "a Tangent and a Secant".

The formulas for all THREE of these situations are the same:

Angle Formed Outside = $\frac{1}{2}$ **Difference** of Intercepted Arcs

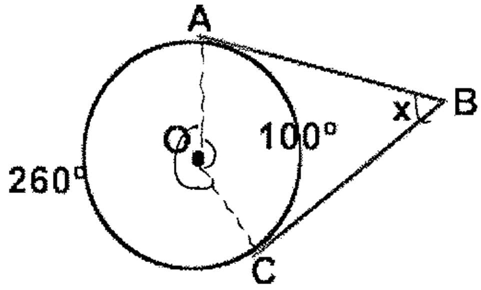
(When subtracting, start with the larger arc.)

Two Tangents:

<ABC is formed by two tangents intersecting outside of circle O.

The *intercepted arcs* are minor arc AC and major arc AC. These two arcs together comprise the entire circle.

$$m\angle ABC=\frac{1}{2}(260-100)=80^\circ$$



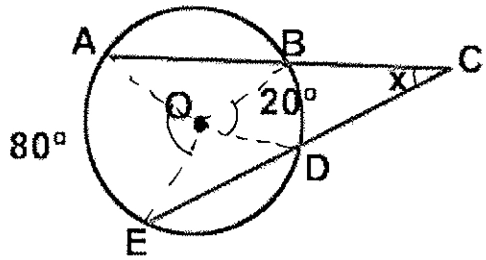
Special situation for this set up: It can be proven that <ABC and central <AOC are supplementary. Thus the angle formed by the two tangents and its first intercepted arc also add to 180°.

Two Secants:

<ACE is formed by two secants intersecting outside of circle O.

The *intercepted arcs* are minor arcs BD and AE.

$$m\angle ACE=\frac{1}{2}(80-20)=30^\circ$$

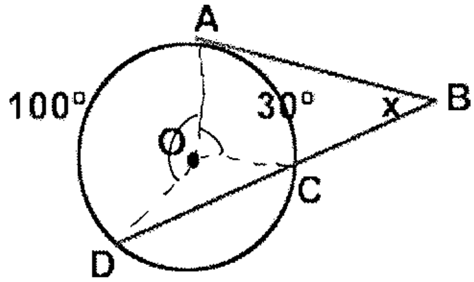


a Tangent and a Secant:

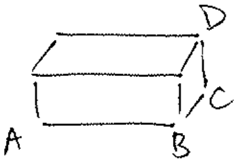
<ABD is formed by a tangent and a secant intersecting outside of circle O.

The *intercepted arcs* are minor arcs AC and AD.

$$m\angle ABD=\frac{1}{2}(100-30)=35^\circ$$



Box



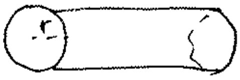
- volume $= AB \cdot BC \cdot CD$
- surface area $= 2 \cdot AB \cdot CD + 2 \cdot BC \cdot CD + 2 AB \cdot BC$

Prism

- volume = base area · height
- surface area = 2 (base area) + (perimeter of base)

cylinder

- volume $= \pi r^2 h$
- surface area $= 2\pi r h$



Pyramid

- volume $= \frac{1}{3}$(base)(height).
- surface area = Base area + sum of areas of all △ faces.

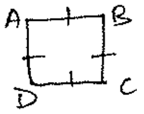
cones

- volume $= \frac{1}{3} \pi r^2 h$
- surface area $= \pi r^2 + \pi r$

sphere

- volume $= \frac{4}{3} \pi r^3$
- surface area $= 4\pi r^2$

Square

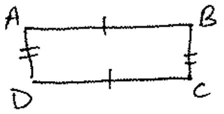


- Area $= AB^2$
- Perimeter $= 4AB$
- Diagonal $= \sqrt{2}\,AB$
- Diagonals bisect

ellipse

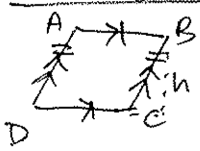
- Area $= \pi r_1 r_2$

Rectangle



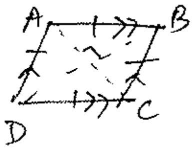
- Area $= AB \cdot BC$
- Perimeter $= 2AB + 2BC$
- Diagonals bisect

Parallelogram



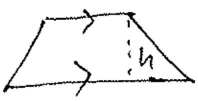
- Area $= DC \cdot h$
- Perimeter $= 2AB + 2BC$
- opp sides & opp ∡ are equal
- opp sides are parallel.

Rhombus



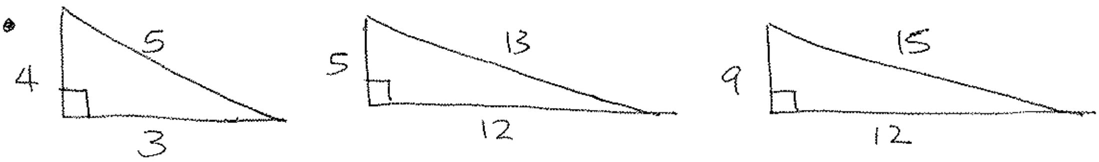
- Area $= \dfrac{AC \cdot BD}{2}$
- perimeter $= 4AC$
- Diagonals bisect & intersect at $90°$
- All ∡s & sides are equal.

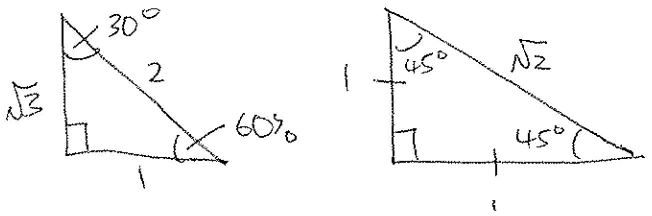
Trapezoid



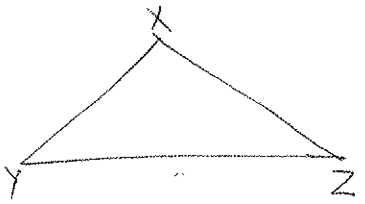
- Perimeter.
- Area $= \frac{1}{2}$ (height · sum of parallel sides)

Triangles

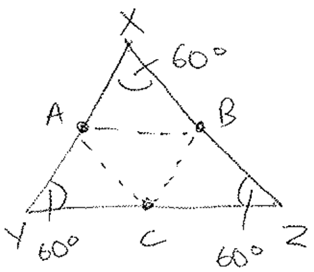




- similar $\triangle$s : correspond^g $\measuredangle$ are equal.
  " sides are proportional.
  areas are in proportion : (ratio of correspond^g lengths)^2



- $YZ^2 = XY^2 + XZ^2$
- $(YX - YZ) < XZ < (YX + YZ)$
- largest $\measuredangle$ opp largest side; same w smallest
- every $\triangle$ has at least 2 acute $\measuredangle$s



- centroid: pt of intersect^n of all 3 bisectors (centre of $\triangle$)
- bisector of every $\measuredangle$ is $\perp$ to opp side & bisects it
- $AB = \frac{1}{2} YZ$
- $XY = XZ = YZ$ (equilateral $\triangle$)
- $ABC = AYC = BCZ = XAB$
- Area of $XYZ = \frac{\sqrt{3} \cdot YZ^2}{4}$

$-XC = \sqrt{YZ^2 + \left(\frac{YZ}{2}\right)^2}$
$= \frac{\sqrt{3}YZ^2}{4}$

- Area of any $\triangle$ $= \sqrt{S(S-XY)(S-XZ)(S-YZ)}$ where $S = \frac{XY+XZ+YZ}{2}$.

- Right $\measuredangle$ $\triangle$ $\Rightarrow$ sum of 2 angles = 3rd angle.

Polygons

- measure of $\measuredangle = 180 - \frac{360}{n}$ where n = no. of sides of polygon.
- sum of int. $\measuredangle$s $= 180(n-2)$
- no. of diagonals $= \frac{1}{2}n(n-3)$
- no. of triangles $= n-2$
- area $= \frac{1}{2}n \sin \frac{360}{n} \cdot s^2$ where s = length fr. centre to corner

clocks

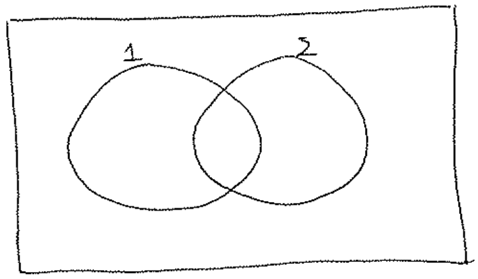
$\measuredangle = \left| \frac{60H - 11M}{2} \right|$ where H = value of hour hand
M = value of min hand.

- $P(\text{not } A) = 1 - P(A)$
- $P(A \cap B) < P(A)$

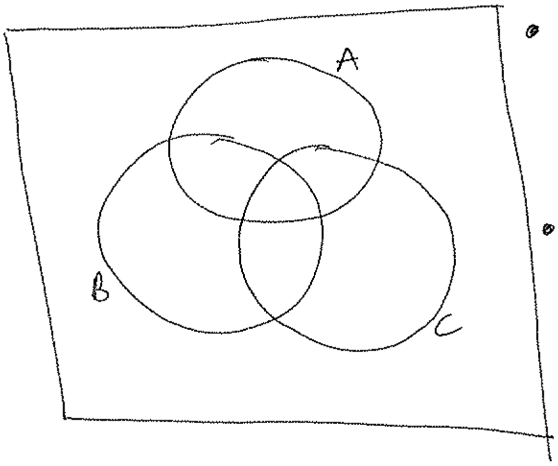
  $P(A \cup B) > P(A)$

  $P(A \cap B) < P(A \cup B)$

  $P(A \cup B) > P(A) + P(B)$

- 

  $Grp_1 + Grp_2 + \text{Neither} - \text{Both} = \text{Total}.$

- 

- $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cup B \cup C)$
- No. of ppl in 1 set $= P(A) + P(B) + P(C) - 2P(A \cap B) - 2P(B \cap C) + 3P(A \cap B \cap C)$
- No. of ppl in 2 sets $= P(A \cap B) + P(A \cap C) + P(B \cap C) - 3P(A \cap B \cap C)$
- No. of ppl in 3 sets $= P(A \cap B \cap C)$
- No. of ppl in 2 or more sets $= P(A \cap B) + P(A \cap C) + P(B \cap C) - 2P(A \cap B \cap C)$

Probability & Permutations

$$P(E) = \frac{n(E)}{n(S)} \quad \text{ie} \quad \frac{\text{event}}{\text{total}}.$$

eg. prob. tt 2 cards drawn out of 52 cards w/out replacement are Qs

$$= \frac{{}^4P_2}{{}^{52}P_2}$$

eg. form a 3 digit no. from digits 1–7 w value ½ 100 & 500

$= $ 1st digit (1–4) ie $4 \cdot \underbrace{{}^6P_2}_{\text{2nd \& 3rd digits}} = 120$

eg. If letters triangle are rearranged randomly, what is the prob tt 1st letter is A.

$$= \frac{1 \cdot {}^7P_7}{{}^8P_8}$$

eg. Pr(5 cointosses) to get 2T & 3H.

$$= \frac{5!}{2!\,3!} \cdot \left(\tfrac{1}{2} \cdot \tfrac{1}{2} \cdot \tfrac{1}{2} \cdot \tfrac{1}{2} \cdot \tfrac{1}{2}\right) = 5/16.$$

eg. Pr (no. amongst 1st 1000 integers is divisible by 8) = 1/8.

Set theory

- Union : A $\{1,2\}$ B $\{3,4\}$

  $A \cup B = \{1,2,3,4\}$

- Intersection: $A \cap B = \{2\}$ when A $\{1,2\}$ B $\{2,3,4\}$

SD of Prob.

- $x_1, x_2, x_3 \ldots x_n$ are n draws fr. random sample

  (a) mean = $\frac{\text{sum }(x_1 \ldots x_n)}{n}$

  (b) variance $= \frac{1}{n}[(x_1-m)^2 + (x_2-m)^2 + \ldots (x_n-m)^2]$

  (c) sd. $= V^{(1/2)}$

Binomial Probability

$P(X=r)\ p^r q^{n-r} \cdot {}^nC_r$ when n: total no. of trials

r: no. of successes

p = prob. of success

q = prob. of failure

Distributing things amongst partitions

Distributing n things among r partitions $= r^n$

- 5 boys & 5 girls stand in a line such th no 2 girls stand next to each other. In how many ways can ē line be formed?

$6 * 5! * 5!$

eg. how many permutat?s for q&e next to each other in square.

$= 5! \cdot 2 = 240$ ie qe suar + eq suar.

- when n dice ($n>1$) are rolled simultaneously, no. of outcomes in which all n dice show ē same no. = 6, irrespective of value of n.

- when n coins ($n>1$) are tossed simultaneously, no. of outcomes in which all n coins turn up as H/T = 1, irrespective of value of n.

- choose 3S & 1J out of 6S & 4J – how many sets of 3S&1J are there

Ans: ${}^6C_3 \cdot {}^4C_1$

- choose either 3R or 3B when tak? 4 types out of 4R & 4B

w̄out replacement: $\frac{{}^4C_3 \cdot {}^4C_1 \cdot 2}{{}^8C_4}$

w̄ replacement :

Ratios

eg. Ratio of m : w is 5 : 4. If 9 more women join, ratio is 10 : 11.
How many women were there originally

$$\frac{5x}{4x+9} = \frac{10}{11} \longrightarrow x = 6$$

Inverse proportionality : 2 variables are inversely proportional when the increase in value of 1 causes a proportional decrease in value of other.

eg.

	s	t	d
A	5m/hr	$\frac{d}{5}$	d
P	$\frac{4}{5}(5)$	$\frac{5}{4}(\frac{d}{5})$	d

for fixed d, s & t are inversely proportional.

Conversion

16 ounces = 1 pound.

2000 pounds = 1 ton

2 cups = 1 pint

2 pints = 1 quart

4 quarts = 1 gallon

12 inches = 1 foot

3 feet = 1 yard

1,760 yards = 1 mile

5,280 feet = 1 mile.

Coins

1 nickel = 5¢

1 dime = 10¢

1 quarter = 25¢

1 half = 50¢

1 dollar = 100¢

- convert coin q^ns into value (fr. no.).

	dimes	quarters	total.
number	D	20-D	20
value.	10D	25(20-D)	305.

Inequalities Absolute

- Remember tt ×/÷ by -ve no. changes ē sign!

- $|a+b| < |c+d|$

$(a+b)^2 < (c+d)^2$

⋮

- $1 \leq |x+3| < 5$

⋮

$-1 \leq x+3 \leq 1$ & $-5 < x+3 < 5$.

- $\frac{1}{a} < \frac{1}{b} \Rightarrow a > b$ eg $\left(\frac{1}{2}\right)^x < \frac{1}{100}$ ie $2^x > 100$

eg. use limits to work out q^ns: $x^2 < 2x < \frac{1}{x}$, $x > 0$.

$$\left.\begin{array}{l} x^2 < 2x \Rightarrow x < 2 \\ 2x < \frac{1}{x} \Rightarrow x < \sqrt{2}/2 \\ x^2 < \frac{1}{x} \Rightarrow x < 1 \end{array}\right\}$$

∴ sol^n lies in ē range $0 < x < \sqrt{2}/2$.

Rates

Yr 1 : a
2 : b
3 : c

$\frac{b-a}{a} \cdot 100 = x\%$

$\frac{c-b}{b} \cdot 100 = y\%$

$\frac{c-a}{a} \cdot 100 = z\%$

inverse variation

5 cats : 4 days : 1 can of food.

8 cats : $\frac{4 \times 5}{8}$ days : 1 can of food.

- Yr 1 : $\downarrow x\%$
  2 : $\downarrow y\%$
  3 : $\downarrow z\%$
  4 : Population, P

Population$_1$ = $\frac{P \cdot 100 \cdot 100 \cdot 100}{(100-x)(100-y)(100-z)}$

- Yr 1 : P (populatn)
  2 : $\downarrow x\%$
  3 : $\downarrow y\%$
  4 : $\downarrow z\%$

Populatn$_4$ = $\frac{P \cdot (100-x)(100-y)(100-z)}{100 \cdot 100 \cdot 100}$.

- Ave speed $= \frac{\text{Total Dist}}{\text{Total Time}}$.

- Equal Dist ; Diff speed $\longrightarrow$ Ave speed $= \frac{2ab}{a+b}$ (harmonic mean)

- Diff Dist ; Equal Time $\longrightarrow$ Ave speed $= \frac{a+b}{2}$

<u>work</u>

- Fixed job, $\frac{1}{r} + \frac{1}{s} = \frac{1}{t}$ where r & s are rates per 1 piece of work for A & B persons separately. t is combined rate when working together.

- A : B
  $x : y$ (work)
  $\frac{1}{x} : \frac{1}{y}$ (work rate)
  $y : x$ (distributⁿ of wages)

- If "a" men of "b" women can do a piece of work in x days, for "m" men & "n" women together, they can finish in $\frac{(abx)}{(an+bm)}$ days (can be derived).

- If A is x times more efficient than B, & both can finish work in y days together, Time taken by A $\Rightarrow y = \frac{(x+1)}{x}$; B $\Rightarrow y(x+1)$

- If A & B can finish ē work in x & ax days respectively, i.e. A is a more efficient than B, working together they can finish in $\left(\frac{xy}{y-x}\right)$ days

- If A & B working together can complete work in x days ; B alone in y days, A alone can complete in $\frac{xy}{(y-x)}$ days.

Exponentials

- $X^a \cdot X^b = X^{(a+b)}$
- $(X^a)^b = X^{ab}$
- $X^c / X^d = X^{(c-d)}$
- every no. raised to power of 5 has no. itself as unit digit

Roots

- $(X\sqrt{Y})(W\sqrt{Z}) = X \cdot W\sqrt{Y \cdot Z}$
- $\dfrac{X\sqrt{Y}}{W\sqrt{Z}} = \dfrac{X}{W}\sqrt{\dfrac{Y}{Z}}$
- consider both $\pm$ roots of $\sqrt{X}$

Difference in squares

- $X^2 - Y^2 = (X+Y)(X-Y)$
- $X - Y = (\sqrt{X} + \sqrt{Y})(\sqrt{X} - \sqrt{Y})$
- eg. $X^{22} - Y^{18} = (X^{11} + Y^9)(X^{11} - Y^9)$

$$\frac{3}{\sqrt{6}+\sqrt{5}} = \frac{3(\sqrt{6}-\sqrt{5})}{(\sqrt{6}+\sqrt{5})(\sqrt{6}-\sqrt{5})}$$

$$= 3(\sqrt{6}-\sqrt{5}) / 1$$

Ratios

- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

  $\therefore = \dfrac{(a+c+e\ldots)}{(b+d+f\ldots)}$

- If $\frac{a}{b} = \frac{c}{d}$

  $\therefore$ – $\frac{b}{a} = \frac{d}{c}$

  – $\frac{a}{c} = \frac{b}{d}$

  – $\dfrac{(a+b)}{b} = \dfrac{(c+d)}{d}$

  – $\dfrac{(a-b)}{b} = \dfrac{(c-d)}{d}$

  – $\dfrac{(a+b)}{(a-b)} = \dfrac{(c+d)}{(c-d)}$

compound interest

- money gets doubled in $70/r$ yrs

  ie. $P(1+\frac{r}{100})^N = 2P$

  when $N = 70/r$

Profit / Loss

- SP of 2 articles are equal; 1 sold at π of $p\%$, other at loss of $p\%$.
  2 trades $\Rightarrow$ Net Δ = loss $\frac{p^2}{100}\%$. (because costs are diff)

- cost of 2 articles equal; 1 sold at π of $p\%$, other at loss of $p\%$.
  2 trades $\Rightarrow$ Net Δ = 0 (no π/loss).

consecutive numbers

- Ave of concecutive nos $= \dfrac{\text{smallest no.} + \text{largest no.}}{2}$
  - ave of even no. of concecutive nos ≠ integer
  - " " odd " " " " = odd integer

- Sum of consecutive nos = average × no. of terms

$$= \left(\frac{\text{smallest no.} + \text{largest no.}}{2}\right)\left(\text{largest no.} - \text{smallest no.} + 1\right)$$

- No. of terms in series = largest no. − smallest no. + 1

- $1+2+3 \ldots + n = \dfrac{n(n+1)}{2}$
- sum of squares of 1st n natural nos $= \dfrac{n(n+1)(2n+1)}{6}$
- sum of cubes of 1st n natural nos $= \left[\dfrac{n(n+1)}{2}\right]^2$
- sum of 1st n odd nos $= n^2$ ie. $2n+1, 2n+3, 2n+5 \ldots$
- sum of 1st n even nos $= n(n+1)$ ie. $2n, 2n+2, 2n+4 \ldots$

If n is even,
no. of odd/even nos fr. 1−n $= n/2$

If n is odd,
no. of odd nos fr. 1−n $= \dfrac{(n+1)}{2}$
" " even " " " " " " $= \dfrac{(n-1)}{2}$

[redacted]

- AP
  - nth term : $T_n = a + (n-1)d$
  - Sum of n terms : $S_n = \frac{n}{2}[2a + (n-1)d]$
  - a : 1st term ; d : common diff.
  - If a, b, c are consecutive terms in an AP, $2b = a + c$

- GP
  - nth term : $T_n = a[r^{n-1}]$
  - sum of n terms : $S_n = a\left[\dfrac{r^{n-1}}{r-1}\right]$ if $r > 1$

$$= a\left[\frac{1-r^{n-1}}{r-1}\right] \text{ if } r < 1$$

  - sum of infinite terms: $S_\infty = \dfrac{a}{1-r}$ if $r < 1$
  - If a, b, c are 3 consecutive integers in a GP, $b^2 = ac$

Remainders

- $\frac{X}{Y}$ r Z i.e. X is multiple of Y + Z

  Y ≥ Z always to get remainder of Z

- $\text{Remainder}_1 + \text{Remainder}_2 - \text{Divisor} = \text{Remainder } 3$.

- How many nos. up to 100 are divisible by 6?

  $\frac{100}{6} = 16 \text{ r } 4$ ∴ 16 nos. within 100 divisible by 6.

- If ē diff % largest & smallest divisor of a no. is X, no. is X + 1

- Eg if s & t are +ve integers such tt $s/t = 64.12$, which could be ē R when s/t?

  → $s/t = 64.12 \Rightarrow s = 64t + \frac{12}{100}t$ ⇒ For t to be an integer, $s \cdot 100/12$ must be one.

Characteristics

Even	+	Even	=	Even
Odd	+	Odd	=	Even
Even	+	Odd	=	Odd.
Even	×	Even	=	Even
Odd	×	Odd	=	Odd.
Even	×	Odd	=	Even.

- If $a - b$ = odd ⇒ $a + b$ = odd.
- $a^n + b^n \div (a+b)$ if n is odd.
  $\cancel{\div}\ (a+b)$ if n is even.
- $a^n - b^n \div (a-b)$ if n is odd/even
  $\div\ (a+b)$ if n is even.

Remainders

- $\frac{a^n}{a+1}$ if n = odd, r = a
  n = even, r = 1

- $\frac{(a+1)^n}{a}$ always r = 1

- When a no. is successively divided by 2 divisors, d_1 & d_2, and 2 remainders r_1 & r_2 are obtained, the remainder tt will be obtained by ē pdt of d_1 & d_2 is given by $d_1 r_2 + r_1$, where d_1 & d_2 are in ascendg order respectively & r_1 & r_2 ē respective remainders

MULTIPLES

- 2 : last digit is even
  3 : sum of digits is multiple of 3
  4 : last 2 digits is multiple of 4
  5 : last digit is 5 or 0
  6 : sum of digits is multiple of 3 + last digit is even
  9 : sum of digits is multiple of 9
  10 : last digit is 0
  12 : sum of digits is multiple of 3 + last 2 digits is multiple of 4

- If 2x is multiple of y, $2x/y$ is an integer
- If P = pdt of 1st x integers, multiples of P must have same prime factors as P. Nos. not multiples of P will have nos. that are not prime factors of P.

- pdt of any 3 consecutive integers is divisible by 6 (or 2 and 3)
  " " " 4 " " " " " 24
- square of any even no. divisible by 4
  " " " odd " " " " gives remainder of 1
- any perfect square can be represented in ē form 4n or 4n+1

- 7 : Double last digit & subtract fr. no. If ans is divisible by 7 (inc 0) then no. is also
- 8 : If last 3 digits form no. divisible by 8, then no. is also
- 11 : Alt add & subtract digits fr. left to right. If result (inc 0) is div. by 11, no. is also. Eg 387421 ie 3 − 8 + 7 − 4 + 2 − 1 = ?
- 13 : Delete ē last digit from ē no, then subtract 9 x ē deleted digit fr. ē remaining no. If what is left is div by 13, so is ē org. no.