



Shute

GRE

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Maths

Shute

Geometry

* Types of Angles -

1) Acute 0° to 90°

2) Right 90°

3) Obtuse 90° to 180°

4) Straight 180°

5) Reflex 180° to 360°

6) Complete 360°

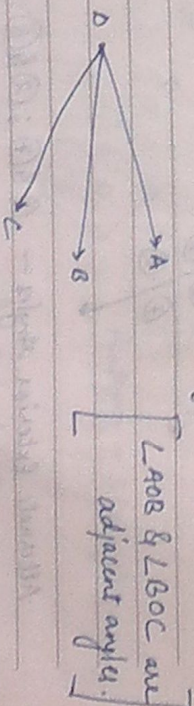
* Complementary Angles \rightarrow Sum up to 90° $\{ \angle A + \angle B = 90^\circ \}$

* Supplementary Angles \rightarrow Sum up to 180° $\{ \angle A + \angle B = 180^\circ \}$

* Adjacent Angles -

1) Share common side & a common vertex.

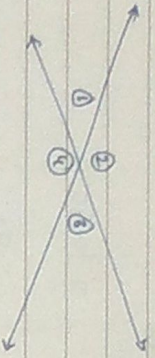
2) Don't share any interior points (or) are angle not contained in other angle.





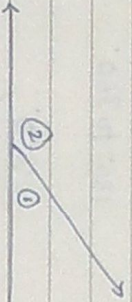
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Vertical Angles - Angles opposite to each other. They are always equal in measure.

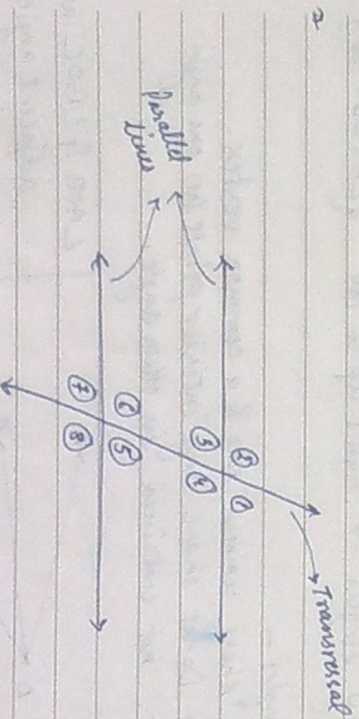


① & ③ } Vertical Angles; Also, ① = ③ & ② = ④
② & ④ }

* Linear Pairs (on a line)



① & ② are linear pairs.
Also, $L1 + L2 = 180^\circ$



Alternate Exterior Angles - ① & ⑦; ② & ⑥. Also, ① = ⑦; ② = ⑥.

Alternate Interior Angles - ③ & ⑤; ④ & ②. Also, ③ = ⑤; ④ = ②.



Shade

Corresponding Angles - ② & ⑥; ① & ⑤

Also, ② = ⑥ & ① = ⑤.

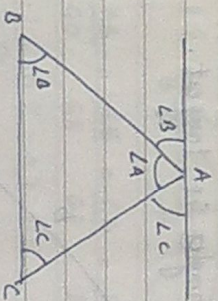
* **Transversal** is a line intersecting two or more lines & lines may or may not be parallel.

Polygons -

Sum of Interior Angles = $(n-2) \times 180^\circ$ { $n \geq 3$ }

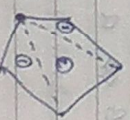
Sum of Exterior Angles = 360° - Always taken in one direction.

* Why sum of Exterior Angles of a Δ equals 180° ?



So, $LA + LB + LC = 180^\circ$

* (Kisi bhi polygon me $(n-2)$ triangles hote hain.)



Pentagon \rightarrow 3 triangles.

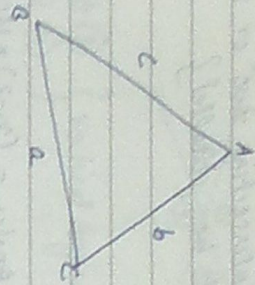
* **Regular Polygon** \rightarrow 1) All sides equal in length.

2) All angles equal in measure.



* A circle is a polygon with infinite sides & length of each side is infinitesimally small.

TRIANGLES -



[At least two angles of a Δ must be acute.]

$$a+b > c$$

$$b+c > a$$

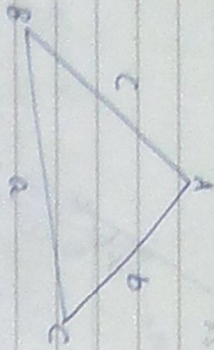
$$a+c > b$$

$$a-b < c$$

$$b-c < a$$

$$c-a < b$$

* Side opposite to largest angle is longest.



$$\angle A > \angle C > \angle B \Rightarrow a > c > b.$$



Categories of Δ 's

White

Based on Sides

- 1) Scalene (All sides diff.)
- 2) Isosceles (Two sides equal)
- 3) Equilateral (All sides equal)

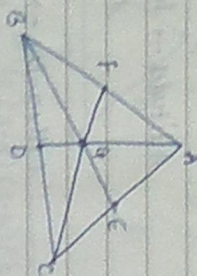
Based on Angles

- 1) Acute Angled (All angles acute)
- 2) Right Angled (One angle 90)
- 3) Obtuse Angled (One angle obtuse)

* Point and lines in a triangle -

1) Centroid & Median

$$\left. \begin{array}{l} BD = DC \\ AF = FA \\ AE = EC \end{array} \right\} \rightarrow$$



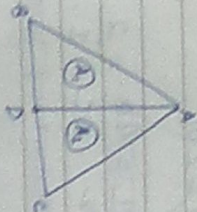
* Median \rightarrow divides the side of Δ into halves.

* Centroid \rightarrow PoI of medians.

$$\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = \frac{2}{1}$$

* Median also divides the area of Δ into two equal parts.

$$\frac{A_1}{A_2} = \frac{BD}{DC} = \frac{1}{1}$$



* Apollonius Theorem for Medians $\rightarrow AB^2 + AC^2 = 2[AD^2 + BD^2]$

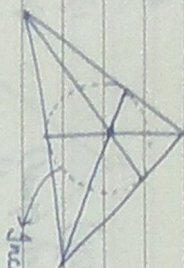


Angle Bisectors and Centroid \rightarrow

State

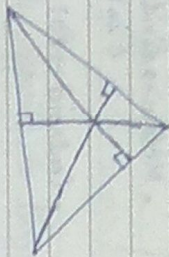
Angle bisector \rightarrow divides the angles into 2 equal parts.

Centroid \rightarrow POI of AB.



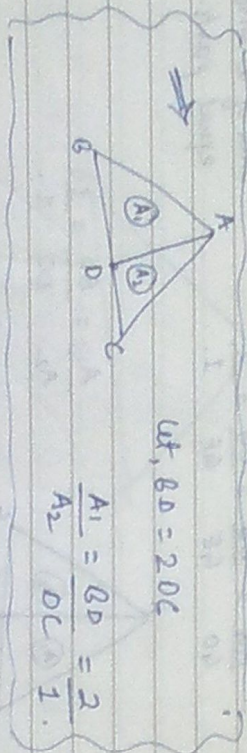
Radius of Incircle $= r$; $r = \frac{\text{Area of } \Delta}{s}$ $\left\{ s = \frac{a+b+c}{2} \rightarrow \text{Semiperimeter} \right\}$

Altitude & Orthocentre $-$



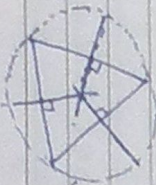
Altitudes \rightarrow height of Δ , perpendiculars dropped from vertices to opposite side.

Orthocentre \rightarrow POI of Altitudes.



$$\frac{A_1}{A_2} = \frac{BD}{DC} = \frac{2}{1}$$

Perpendicular Bisector & Circumcentre \rightarrow



Radius of Circumcircle $= R$; $R = \frac{a \cdot b \cdot c}{4 \text{ Area } \Delta}$

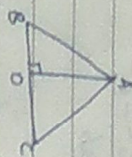
State

\Rightarrow

$$\text{Area } \Delta = \frac{abc}{4R}$$

Area of Triangle $-$

1)

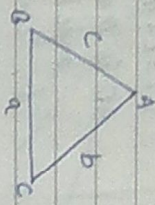


$$AD = h$$

$$BC = b$$

$$\text{Area} = \frac{1}{2}bh \quad \langle \text{half} \cdot \text{base} \cdot \text{height} \rangle$$

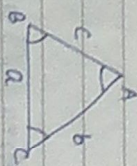
2)



$$s = \frac{a+b+c}{2}$$

$$\text{Area of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

3)



$$\text{Area of } \Delta = \frac{1}{2}ab \sin C = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B$$

Sine formula / Law's Theorem $\Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$.

$$\sin A \quad \sin B \quad \sin C$$

where, $R =$ radius of circumcircle.



Date

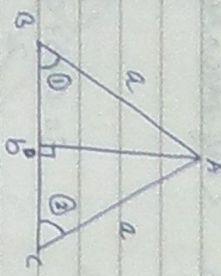
1) cosine formula -

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{b^2 + a^2 - c^2}{2ab}$$

2) ISOSCELES TRIANGLES -

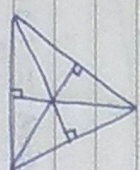


$$L1 = L2 \Leftrightarrow AB = AC$$

AD is Median, Altitude,
angle bisector & \perp bisector.
So, Centroid, orthocentre,
incentre & circumcentre
coincide & lie on same line \angle

$$\text{Area of } \Delta = \frac{b}{4} \sqrt{4a^2 - b^2}$$

3) Equilateral Δ -



$$\text{Area} = \frac{\sqrt{3}}{4} a^2$$

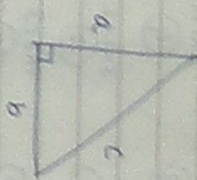
$$\text{Height} = \frac{\sqrt{3}}{2} a$$

Median, Altitude, Angle bisector & \perp bisectors are all same, so,
centroid, orthocentre, incentre & circumcentre coincide & lie on same
line.



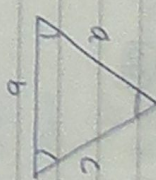
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PYTHAGORAS THEOREM -



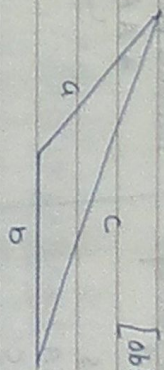
$$a^2 + b^2 = c^2$$

[Right Δ]



[Acute angled Δ]

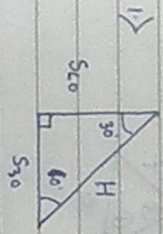
$$a^2 + b^2 < c^2$$



[Obtuse angled Δ]

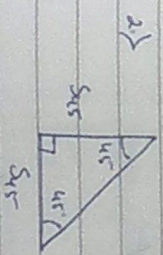
$$a^2 + b^2 > c^2$$

4) SPECIAL Δ 'S -



[30-60 Δ]

$$S_{30} = \frac{H}{2} ; S_{60} = \frac{\sqrt{3}}{2} H$$



[45-45 Δ]

$$S_{45} = \frac{H}{\sqrt{2}}$$

Other Pythagorean Triplets $\rightarrow 8, 15, 17$ & $7, 24, 25$

Date



Pythagorean Triplets \rightarrow { Also called odd triplets }

3, 4, 5 \rightarrow (1) \rightarrow (3)

3x+1 (4) 4x+1 (5)

5, 12, 13 \rightarrow (2) \rightarrow (5)

5x+2 (12) 12x+1 (13)

7, 24, 25 \rightarrow (3) \rightarrow (7)

7x+3 (24) 24x+1 (25)

9, 40, 41 \rightarrow (4) \rightarrow (9)

9x+4 (40) 40x+1 (41)

[8, 15, 17]

\rightarrow Congruent & Similar Δ 's \rightarrow

Postulates for Congruency:-

- 1) SSS
- 2) SAS
- 3) ASA
- 4) RHS

Postulates for Similar Δ 's:-

- \rightarrow AA
- \rightarrow AAA



So, By Similarity $\rightarrow \frac{a}{d} = \frac{b}{e} = \frac{c}{f}$

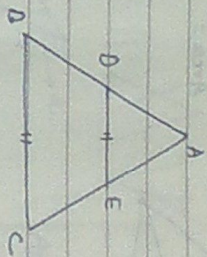


Each \angle is \angle interior angle is equal to sum of opposite interior angles



BASIC PROPORTIONALITY THEOREM-

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$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

ΔABC & ΔADE are similar

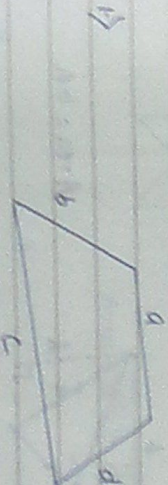
* Sin, Cos & tan in diff. Quadrants-

sin \oplus	cos \oplus	tan \oplus
sin \ominus	cos \ominus	tan \ominus

{ Add signs to coffee }

QUADRILATERALS

Polygon with 4 sides. \rightarrow Sum of Interior Angles = 360°
Sum of Ext. Angles = 360°



$$\text{Area} = \frac{(5-a)(5-b)(5-c)(5-d)}{2}, \quad E = \frac{a+b+c+d}{2}$$

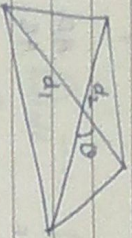


$$\text{Area} = \frac{d}{2} (h_1 + h_2)$$



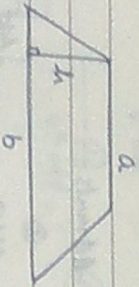
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③



$$\text{Area} = \frac{d_1 \cdot d_2 \cdot \sin \theta}{2}$$

2 Trapezium -

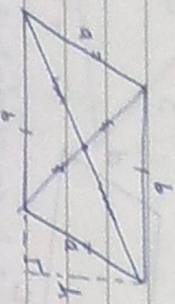


$$\text{Area} = \frac{1}{2} (a+b) h$$

Isosceles Trapezium \rightarrow 1) Non-parallel sides have equal length.

- 2) Length of diagonals is same
- 3) Diagonals intersect each other in same ratio.

1 Parallelogram -



$$\text{Area} = bh$$

1) Length of Diagonals is not same.

2) But, diagonals bisect each other.

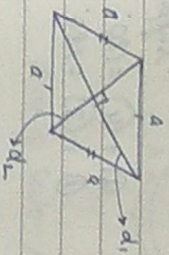


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1 Rhombus -

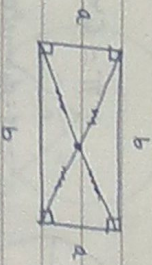


- 1) Diagonals \perp bisectors.
- 2) Not of equal length.



$$\text{Area} = \frac{d_1 d_2}{2}$$

2 Rectangle -



- 1) Diagonals bisect.
- 2) Equal in length (diagonals)

$$\text{Area} = ab ; \text{Perimeter} = 2(a+b)$$

3 Square -



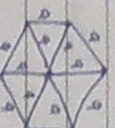
- 1) Diagonals \perp bisectors.
- 2) Equal in length (diagonals)

$$\text{Area} = a^2 = \frac{d^2}{2}$$

4 Regular Hexagon - 1) Polygon with 6 sides

2) All sides & angles are equal.

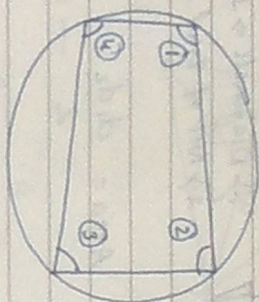
$$\text{Area} = \frac{3\sqrt{3}}{2} a^2$$



[6-Equilateral Δ of side a formed]

Cyclic Quadrilateral -

Date

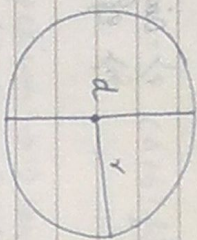


opposite angles are supplementary.

$$\angle 2 + \angle 4 = 180^\circ$$

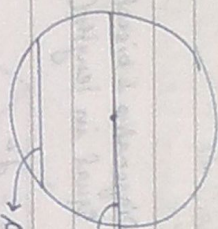
$$\angle 1 + \angle 3 = 180^\circ$$

CIRCLES -

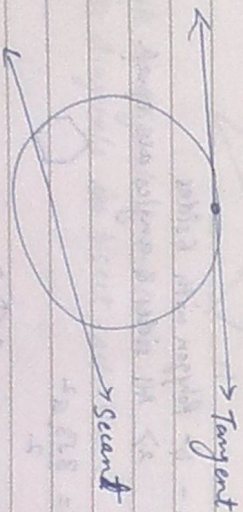


$$\pi = \frac{\text{Circumference}}{\text{Diameter}} = \frac{22}{7}$$

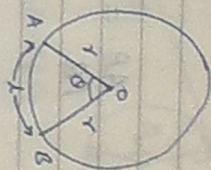
d: diameter, r: radius
[d = 2r]



diameter (longest chord)



Date



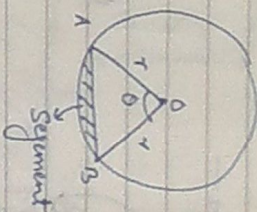
\widehat{AB} = Arc AB

θ = measure (in) of arc = m \widehat{AB}

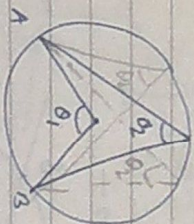
L = length (L) of arc = $L \widehat{AB}$

Area enclosed by \widehat{AB} = Sector.

$$\text{Area of Sector } AB = \frac{\theta}{360} \times \pi r^2 ; L \widehat{AB} = \frac{\theta}{360} \times 2\pi r$$



$$\begin{aligned} \text{Area of Segment} &= \text{Area of Sector } AB - \text{Area of } \triangle OAB \\ &= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin \theta \end{aligned}$$



$$\theta_1 = 2\theta_2$$

θ_1 = Central Angle

θ_2 = Inscribed Angle.

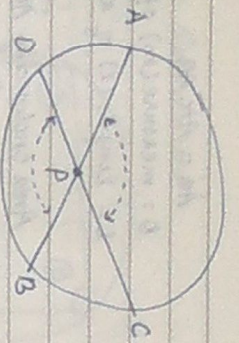
* Angle inscribed in same segment are equal.

Angle inscribed in semicircle is 90°





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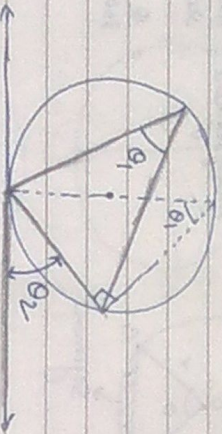


$$\frac{AP}{PB} = \frac{CP}{PD}$$

$$\frac{AP}{PB} = \frac{CP}{PD}$$

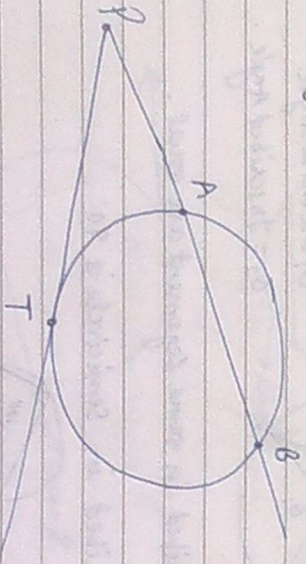
THEOREMS -

1) Alternate Segment Theorem -



$$\theta_1 = \theta_2$$

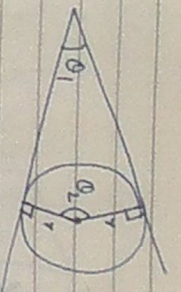
2) Tangent Secant Theorem -



$$PT^2 = PA \times PB$$



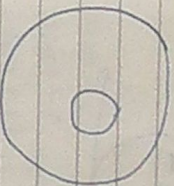
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$$\theta_2 = 2\theta_1$$

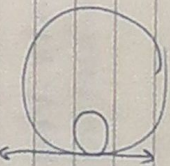
Common TANGENTS -

1)



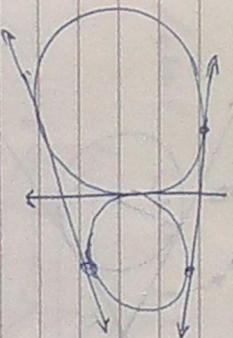
No common tangent

2)



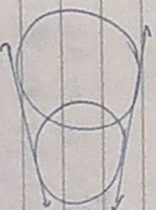
One common tangent

3)



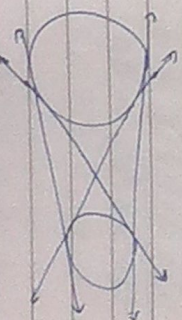
Three common Tangents.

4)



Two common Tangents.

5)

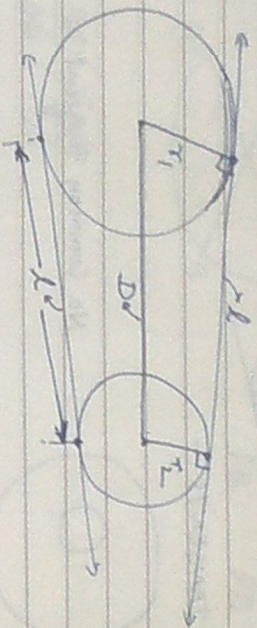


Four common Tangents.



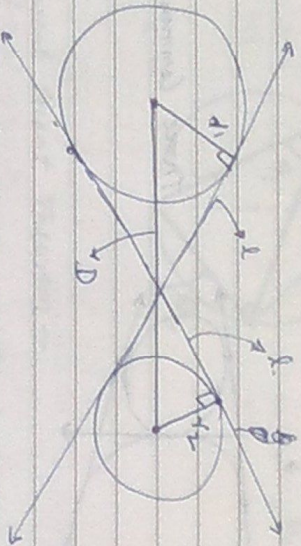
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DIRECT COMMON TANGENT -



$$l = \sqrt{D^2 - (r_1 - r_2)^2}$$

TRANSVERSE COMMON TANGENT -



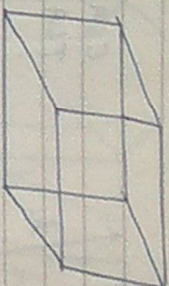
$$l = \sqrt{D^2 - (r_1 + r_2)^2}$$



Shade

3D

* Cube



All sides equal in length.
Let, length = a.

Face - 6

Vertex - 8

Edges - 12

SA $\rightarrow 6a^2$

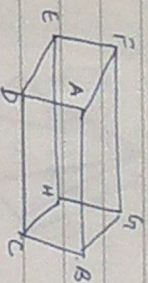
V $\rightarrow a^3$

P $\rightarrow 12a$

Length of Face Diagonal (FD) = $a\sqrt{2}$

Length of Body Diagonal (BD) = $a\sqrt{3}$

* Cuboid



AB = CD = EF = GH

AD = BC

HC = AG

AC = BD

SA $\rightarrow 2(ab + bc + ac)$

V $\rightarrow a \cdot b \cdot c$

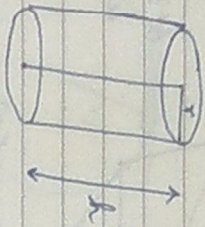
FD $\rightarrow \sqrt{a^2 + b^2}$; $\sqrt{b^2 + c^2}$; $\sqrt{c^2 + a^2}$

BD $\rightarrow \sqrt{a^2 + b^2 + c^2}$



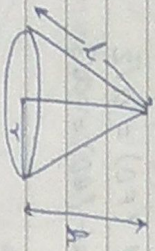
Cylinder { Right Circular Cylinder }

Date



$$\begin{aligned} CSA &\rightarrow 2\pi rh \\ TSA &\rightarrow 2\pi rh + 2\pi r^2 \\ &= 2\pi r(r+h) \\ V &\rightarrow \pi r^2 h \end{aligned}$$

* Cone { Right Circular Cone }



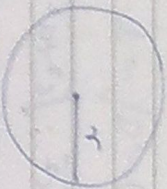
$$\begin{aligned} l &= \text{slant height} \\ &= \sqrt{h^2 + r^2} \end{aligned}$$

$$CSA \rightarrow \pi r l \quad \left\{ \left[\frac{2\pi r + 0}{2} \right] \times l \right\}$$

$$TSA \rightarrow \pi r(l+r)$$

$$V \rightarrow \frac{1}{3} \pi r^2 h$$

* Sphere -



$$\begin{aligned} CSA / TSA / SA &\rightarrow 4\pi r^2 \\ V &\rightarrow \frac{4}{3} \pi r^3 \end{aligned}$$

* Hemisphere -

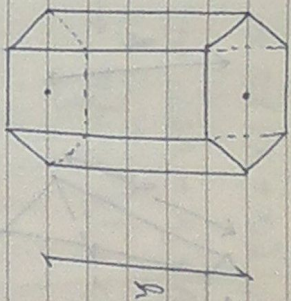


$$\begin{aligned} CSA &\rightarrow 2\pi r^2 \\ TSA &\rightarrow 3\pi r^2 \\ V &\rightarrow \frac{2}{3} \pi r^3 \end{aligned}$$



Prism -

Date



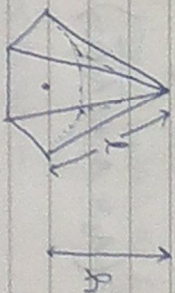
$$\left\{ \begin{aligned} P &= \text{Perimeter} \\ A &= \text{Base Area} \end{aligned} \right\}$$

Lateral Surface Area (LSA) $\rightarrow Ph$

$$TSA \rightarrow LSA + 2A$$

$$V \rightarrow A \cdot h$$

Pyramid -



$$LSA \rightarrow \frac{P \cdot l}{2}$$

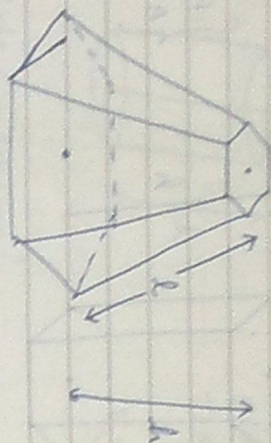
$$TSA \rightarrow LSA + A$$

$$V \rightarrow \frac{1}{3} A \cdot h$$



FRUSTUM OF PYRAMID -

Shade



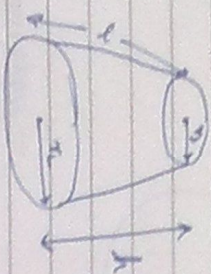
$\left\{ \begin{array}{l} P_s \rightarrow \text{Perimeter of small base} \\ P_L \rightarrow \text{Perimeter of large base} \end{array} \right\}$
 $\left\{ \begin{array}{l} A_s \rightarrow \text{Area of small base} \\ A_L \rightarrow \text{Area of large base} \end{array} \right\}$

$$LSA \rightarrow \left(\frac{P_s + P_L}{2} \right) \cdot l$$

$$TSA \rightarrow LSA + A_s + A_L$$

$$V \rightarrow \frac{h}{3} \cdot (A_s + A_L + \sqrt{A_s A_L})$$

* FRUSTUM OF CONE -



$$LSA \rightarrow \left(\frac{P_s + P_L}{2} \right) \cdot l$$

$$TSA \rightarrow LSA + A_s + A_L$$

$$V \rightarrow \frac{h}{3} (A_s + A_L + \sqrt{A_s A_L})$$



Formulae Reduce form of Frustum of cone -

Shade

$$LSA \rightarrow \pi l (r_1 + r_2)$$

$$TSA \rightarrow LSA + A_s + A_L \rightarrow LSA + \pi r_1^2 + \pi r_2^2$$

$$V \rightarrow \frac{\pi h}{3} (r_1^2 + r_2^2 + r_1 r_2)$$

CONVEX & CONCAVE POLYGON:-

1) CONVEX POLYGON:-



a) All diagonals within the figure.

b) All interior angles less than 180°

2) CONCAVE POLYGON:-
(Concave in)



a) At least one diagonal lies outside the figure.

b) At least one angle is greater than 180°.



Date

No. of sides	Name of Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

WORD PROBLEMS

Date

- 1) Profit & Loss
- 2) Simple Interest & Compound Interest
- 3) Mixture
- 4) Work & Time
- 5) Time, Speed & Distance

PROFIT & LOSS -

$$\begin{array}{l|l} \text{* } SP = CP + P & SP = CP - L \\ CP = SP - P & CP = SP + L \\ P = SP - CP & L = CP - SP \end{array}$$

$$\text{* } P\% = \frac{P}{CP} \times 100 ; \quad L\% = \frac{L}{CP} \times 100$$

$$\text{* } SP = CP \left[1 \pm \frac{x}{100} \right] \quad \left\{ \begin{array}{l} \text{+ve for profit} \\ \text{-ve for loss} \end{array} \right\}$$

* Concept of same selling Price -

 S_1 S_2

$$SP \rightarrow \quad 200 \quad \quad 200$$

$$P/L \rightarrow \quad 10\% \uparrow \quad \quad 10\% \downarrow$$

$$CP \rightarrow \quad \frac{200}{1.1} = 181.81 \quad \quad \frac{200}{0.9} = 222.22$$

$$\text{Total } CP = 404.04$$

$$\text{Total } SP = 400$$

$$\Rightarrow \text{Always a loss is incurred}$$

$$[\text{Loss \%} = \left(\frac{\text{Common Loss \& Gain \%}}{10} \right)^2 = \frac{x^2}{100}]$$



Concept of false weighing:-

Assume $SP = CP$ & profit gained by using false weight

$$\text{Gain \%} = \frac{\text{Error}}{\text{True Value} - \text{Error}} \times 100$$

= Marked / Tagged / Listed / Asking Price :- (MP)

$$SP = MP - \text{Discount} = MP - x\% \text{ of } MP$$

$$SP = MP \left[1 - \frac{x}{100} \right]$$

= Successive Discounts:-

1st $x\%$ discount & then $y\%$ discount.

$$SP = MP \left(1 - \frac{x}{100} \right) \left(1 - \frac{y}{100} \right)$$

Net Discount after successive discounts of $x\%$ & $y\%$:-

$$\Rightarrow x + y - \frac{xy}{100} \left\{ 100 - \left[100 \left(1 - \frac{x}{100} \right) \left(1 - \frac{y}{100} \right) \right] \right\}$$

Simple Interest & Compound Interest:-

Simple Interest:-

$$SI = \frac{PRT}{100}$$



Amount (A) = $P + SI$

$$= P \left(1 + \frac{RT}{100} \right)$$

Compound Interest:-

1) Compounded Annually:-

$$\text{Amount (A)} = P \left(1 + \frac{r}{100} \right)^T$$

$$CI = A - P$$

2) Compounded half yearly:-

$$A = P \left(1 + \frac{r/2}{100} \right)^{2T}$$

3) Compounded Quarterly:-

$$A = P \left(1 + \frac{r/4}{100} \right)^{4T}$$

4) Compounded Monthly:-

$$A = P \left(1 + \frac{r/12}{100} \right)^{12T}$$

Difference in CI & SI for different years on same Principal:-

$$1) \text{ One Year } \Rightarrow CI_1 - SI_1 = 0$$

$$2) \text{ Two Years } \Rightarrow CI_2 - SI_2 = P \left(\frac{R}{100} \right)^2$$

Date

Date



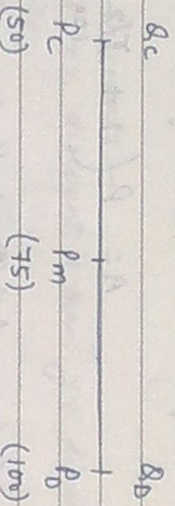
Date

$$3) \frac{\text{True Value}}{CI_3 - SI_3} = \frac{PR^2 (300 + R)}{100^3}$$

$$4) \frac{\text{True Value}}{CI_4 - SI_4} = P \left[6 \left(\frac{R}{100} \right)^2 + 4 \left(\frac{R}{100} \right)^3 + \left(\frac{R}{100} \right)^4 \right]$$

MIXTURES:-

* Price of mixtures always lies between cheaper & dearer.
* Mixture's Price is never less than cheaper & never more than dearer.



$$B_c = \frac{P_d - P_m}{P_d - P_c}$$

CP of unit quantity
of dearer (P_d)

CP of unit quantity
of cheaper (P_c)

Mean Price

(P_m)

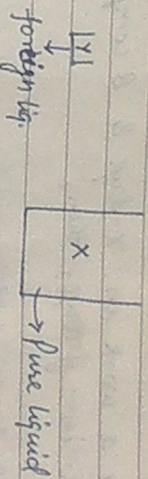
$$(P_d - P_m)$$

$$(P_m - P_c)$$



Date

Mixtures of liquids:-



Suppose a container contains X unit of liquid from which Y unit are taken out & replaced by another liquid. After n -operations, the quantity of pure liquid can be given as:-

$$\text{Quantity of pure liquid (QF)} = X \left(1 - \frac{Y}{X} \right)^n$$

after n -operations

QF = Final Quantity of liquid
QI = Initial Quantity of liquid

$$\text{Also, } \frac{QF}{QI} = \left(\frac{X - Y}{X} \right)^n$$

{ QF may be X or other }

Also, if replacement is first done a -times by y lt vessel, then b -times by w lt vessel & c -times by z lt vessel,

$$\frac{QF}{QI} = \left(\frac{X - y}{X} \right)^a \left(\frac{X - w}{X} \right)^b \left(\frac{X - z}{X} \right)^c$$

WORK & TIME:-

$$\text{Rate of doing work} = \frac{\text{Work Done}}{\text{Time}} \rightarrow R = \frac{W}{T}$$



Wenn $w=1 \Rightarrow R = \frac{1}{T} \quad \& \quad T = \frac{1}{R}$

2. A completes a work in x -days & B completes a work in y -days, then,

1) A & B together done in -

Work in one day = $\frac{1}{x} + \frac{1}{y}$

$$\frac{h\nu}{h + \nu} =$$

$$\text{So, Total Days} = \frac{x^2 y}{x + y}$$

2.) A & B together alternately -

[Lat; A \rightarrow 8 days
B \rightarrow 12 days]

o) A starts the work -

work done @ avg first day = $\frac{1}{8}$

Work done @ end of second day = $\frac{1}{8} + \frac{1}{12}$

$$= \frac{5}{24}$$

So, work done in two days = $\frac{5}{24}$

Ans, 2 days 4 days 6 days 8 days 10 days

$$\frac{5}{24} \quad \frac{10}{24} \quad \frac{15}{24} \quad \frac{20}{24} \quad \left(\frac{25}{24}\right) \rightarrow > 1$$



89, work is completed after 8 days but before 10-days.

② 8 days \rightarrow $W = \frac{20}{24}$

② 9th day A will do the work \rightarrow 40% 20% 20%
25% 5%

Work left to be done = $1 - \frac{20}{24}$ or

$$= \frac{4}{24} = \frac{1}{6}$$

On 9th day A will do $\frac{1}{x}$ work

So, work left for 10th day = $\frac{1}{6} - \frac{1}{8} = \frac{1}{24}$

② 10th day B will come & will do the remaining $\frac{1}{24}$ work

$$\text{Time reqd. by 8 to do } \frac{1}{24} \text{ work} = \frac{1/24}{1/12} = 0.5 \text{ days}$$

So, work will be finished in 9.5 days if A starts the work & A & B work ^{on} alternate days.

b) B starts the work - Again work done in 8 days = 20

Work left to be done = $\frac{1}{6}$

② 4th day B comes and does $\frac{1}{12}$ work, so, work left to be done by A on 10th day = $\frac{1}{6} - \frac{1}{12} = \frac{1}{12}$.

Time taken by A to do $\frac{1}{2}$ work = $\frac{1/2}{1/8} = 0.67$ days. So, total time reqd = 9.67 days



* LOOK EQUIVALENCE CONCEPT -

Date

A \rightarrow x ↑ ; B \rightarrow y ↑ ; C \rightarrow z ↓

$$\text{Rate of filling} = \frac{1}{x} + \frac{1}{y} - \frac{1}{z}$$

$$\text{Rate of emptying} = \frac{1}{z} - \left[\frac{1}{x} + \frac{1}{y} \right]$$

* Time, Speed & Distance -

$$T = \frac{D}{S} ; S = \frac{D}{T} ; T = \frac{D}{S} ; D = S \times T$$

$$\text{for } T = \text{constant} \rightarrow D \propto S \Rightarrow \frac{D_1}{S_1} = \frac{D_2}{S_2} = K$$

$$\text{for } S = \text{constant} \rightarrow D \propto T \Rightarrow \frac{D_1}{T_1} = \frac{D_2}{T_2} = K$$

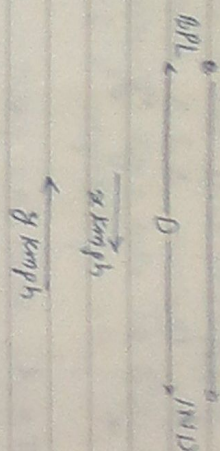
$$\text{for } D = \text{constant} \rightarrow S \propto \frac{1}{T} \Rightarrow S_1 T_1 = S_2 T_2 = K$$

$$1 \text{ km/hr} = \frac{5}{18} \text{ m/s} ; 1 \text{ m/s} = \frac{18}{5} \text{ km/hr}$$

$$\text{Average Speed} = \frac{\text{Total Distance travelled}}{\text{Total time taken}}$$



Date



$$\text{Avg speed} = \frac{d+d}{\frac{d}{x} + \frac{d}{y}} = \frac{2xy}{x+y}$$

* Relative Speed -

1) In same direction - $\xrightarrow{u} \xrightarrow{v}$

$$\text{Relative speed} = |u - v|$$

2) In opposite direction -

a) $\xrightarrow{u} \xleftarrow{v}$

b) $\xleftarrow{u} \xrightarrow{v}$

$$\text{Relative speed} = u + v$$

* TRAINS :- 6 cases possible

1)



$$T = \frac{x}{u}$$



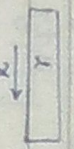
Shute

II)

 \vec{x} \vec{y}

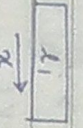
$$T = \frac{l}{x+y}$$

III)

 \vec{y}

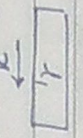
$$T = \frac{l}{x-y}$$

IV)



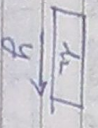
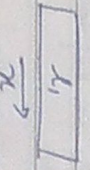
$$T = \frac{l_1 + l_2}{x}$$

V)



$$T = \frac{l_1 + l_2}{x+y}$$

VI)



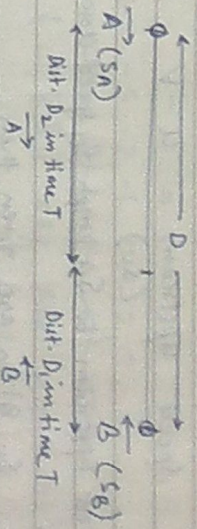
$$T = \frac{l_1 + l_2}{x+y}$$



Shute

If two trains start at the same time from two points A & B towards each other and after crossing they take T_A & T_B hours in reaching B & A respectively.

Then, A 's speed : B 's speed = $\sqrt{T_B} : \sqrt{T_A}$


 T_B
 T_A

$$T = \frac{D_1}{S_B} = \frac{D_2}{S_A}$$

$$T_A = \frac{D - D_2}{S_A} = \frac{D_1}{S_A}$$

$$T_B = \frac{D - D_1}{S_B} = \frac{D_2}{S_B}$$

$$S_B \cdot \frac{T_B}{T_A} = \left(\frac{S_A}{S_B} \right)^2 \Rightarrow \frac{S_A}{S_B} = \sqrt{\frac{T_B}{T_A}}$$

$$\therefore T \propto \frac{1}{S^2} \Rightarrow S \propto \frac{1}{\sqrt{T}}$$



BOTS & STREAMS:-

Shine

Let, Speed of boat = u kmph

Speed of stream = V kmph.

So, Speed downstream = $u + V$
(S_D)

Speed upstream = $u - V$
(S_U)

If Speed upstream be S_U kmph & speed downstream

If S_U & S_D are given, then,

$$\text{Speed of boat} = \frac{S_D + S_U}{2}$$

in still water

$$\text{Speed of stream} = \frac{S_D - S_U}{2}$$



Arithmetic & Fractions:-

Shine

* Real number is any number that can be represented on number line.

→ All numbers are either positive or negative except zero.

→ Integers $\dots -3, -2, -1, 0, 1, 2, 3, \dots$

→ Properties of a, b & c are real numbers.

$$\begin{aligned} 1) \quad a + b &= b + a & \text{Commutative Property} \\ a \times b &= b \times a \end{aligned}$$

2) $a + a - b \neq b - a$ → Subtraction is not commutative.

Only, If $a = b$, then $a - b = b - a$

$$\text{Also, } a \div b \neq b \div a$$

3) Associative Property:-

$$(a + b) + c = a + (b + c)$$

$$(a \times b) \times c = a \times (b \times c)$$

$$+ \quad (a - b) - c \neq a - (b - c)$$

$$\text{a} \quad (a \div b) \div c \neq a \div (b \div c)$$

3) Distributive Property:-

$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac$$



Shane

4)

$$1 \times a = a$$

$$a \div 1 = a$$

$$a \times 0 = 0$$

$$a + 0 = a$$

$$a \div a = 1 \quad \& a \neq 0$$

$$a + (-b) = a - b$$

$$a - (-b) = a + b$$

* Adding positive numbers \rightarrow move right along the number line.

* Subtracting positive numbers \rightarrow move left along the number line.

* Multiplying & Dividing signed numbers:-

$$+ + = +$$

$$+ - = -$$

$$- + = -$$

$$- - = +$$

\rightarrow 2 like signs produce a positive number.

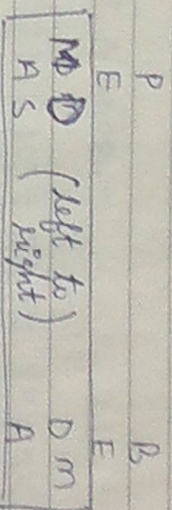
\rightarrow 2 unlike signs produce a negative number.



Shane

COMBINATION OPERATIONS -

\rightarrow Order of Operation = PEMDAS | BEDMAS



\hookleftarrow Evaluate expression from left to right.

\rightarrow Absolute value of x :-

$$|x|$$

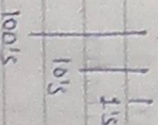
* It is the numbers distance from zero on the number line.

* Absolute value of a number is the simply the positive version of that number. Absolute value is always positive.

$$|-4| = 4, \quad |5| = 5$$

DECIMALS:-

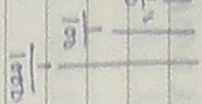
$$743 = 7 \times 100 + 4 \times 10 + 3 \times 1$$





Date

$$82.743 = 8 \times 10 + 2 \times 1 + 7 \times \frac{1}{10} + 4 \times \frac{1}{100} + 3 \times \frac{1}{1000}$$



* Rounding Decimals -

If next digit is 0, 1, 2, 3 or 4 → round down.

If next digit is 5, 6, 7, 8, or 9 → round up.

Eg:-

1) 7.38241 rounded to nearest tenth.

$$7.38241 \approx 7.4 \quad (\text{Round Up})$$

2) 15.02318 rounded to nearest thousandth -

$$15.02318 \approx 15.023 \quad (\text{Round Down})$$

FRACTIONS:-

3 → Numerator

4 → Denominator.

* Equivalent fraction -

$$\frac{1}{2} = \frac{5}{10} = \frac{15}{30}, \quad \frac{7}{9} = \frac{14}{18} = \frac{21}{27}$$



Date

Generate equivalent fraction - multiply or divide the numerator or denominator by same number.

* Lowest Number:-

$$\left(\frac{2}{3} \right) = \frac{10}{15} = \frac{22}{33} = \frac{14}{21} = \frac{90}{135}$$

(lowest term)

* Converting entire fraction/improper fraction to mixed number:-

$$[* \text{ Num} > \text{Deno}]$$

$$\frac{7}{2} = \left(3 \frac{1}{2} \right) \rightarrow \frac{2 \times 3 + 1}{2} \quad \text{mixed no.}$$

* Fraction to Decimal Conversion -

$$* \frac{1}{4} = 0.25 \quad (\text{Treat fraction as division})$$

$$* \frac{5}{6} = 0.8\bar{3}$$

$$* \frac{1}{2} = 0.5$$

$$\frac{1}{6} = 0.1\bar{6}$$

$$\frac{1}{10} = 0.1$$

$$\frac{1}{3} = 0.3\bar{3}$$

$$\frac{1}{7} = 0.1428$$

$$\frac{1}{11} = 0.0909$$

$$\frac{1}{4} = 0.25$$

$$\frac{1}{8} = 0.125$$

$$\frac{1}{5} = 0.20$$

$$\frac{1}{9} = 0.11$$



Date

* Decimal to fraction conversion -

$$0.35 = \frac{35}{100} = \frac{7}{20}$$

$$0.004 = \frac{4}{1000} = \frac{1}{250}$$

* Properties of fraction -

$$n = \frac{n}{1} ; \frac{n}{0} = \text{Not defined}$$

$$\frac{n}{n} = 1 \quad (n \neq 0) ; \frac{1}{a/b} = \frac{b}{a} \quad (a \neq 0, b \neq 0)$$

$$\frac{a \times b}{b} = 1 ; \frac{a}{b} \div \left(\frac{a}{b}\right) = 1 \quad (a \neq 0, b \neq 0)$$

* Bigger Numerator \Rightarrow Bigger value.

$$\frac{3}{11} > \frac{8}{11} \Rightarrow \frac{4}{11} > \frac{3}{11}$$

* Smaller Numerator \Rightarrow Smaller Numerator/Value.* Bigger Denominator \Rightarrow Smaller value.

$$\frac{2}{3} < \frac{2}{4} \Rightarrow \frac{2}{4} < \frac{2}{3}$$

* Smaller Denominator \Rightarrow Bigger value.

Date

* Increase Numerator & Denominator by same amount \rightarrow fraction approaches to 1

$$\frac{2}{7} \xrightarrow{+10} \frac{12}{17} \xrightarrow{+1000} \frac{1012}{1017}$$

closer to 1 more closer to 1.

* PROPERTIES OF FRACTIONS -

$$\rightarrow \frac{abc}{def} = \frac{a \times b \times c}{d \times e \times f}$$

$$\rightarrow \frac{a+b+c}{d+e+f} \neq \frac{a+b+c}{d+e+f}$$

$$\rightarrow \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\rightarrow \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$$

$$\rightarrow \frac{a+b}{c+d} = \frac{a}{c+d} + \frac{b}{c+d}$$

$$\rightarrow \frac{a-b}{c-d} = \frac{a}{c-d} - \frac{b}{c-d}$$

$$\rightarrow \text{If } \frac{a}{b} = \frac{c}{d} \Rightarrow ad = cb$$



PERCENTAGE & RATIOS:-

Date

* Percent = per 100

$$\hookrightarrow 19\% = \frac{19}{100} = 0.19$$

$$\hookrightarrow 0.43\% = \frac{0.43}{100} = 0.0043$$

$$\hookrightarrow 300\% = \frac{300}{100} = 3$$

→ Decimal to Percentage \Rightarrow Move decimal to two places to right.

$$+ 0.00007 = 0.007\% ; + 0.456 = 45.6\%$$

$$+ 3.5 = 350\%$$

→ Percentage to Decimal \Rightarrow move decimal to two places to left.

$$9.63\% = 0.0963 ; 125\% = 1.25$$

→ Fraction to Percentage \Rightarrow Fraction to Decimal to Percentage.

$$\frac{3}{8} = 0.375 = 37.5\%$$



Date

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{1}{3}$	0.3333	33.33%
$\frac{1}{4}$	0.25	25%
$\frac{1}{5}$	0.20	20%
$\frac{1}{6}$	0.1666	16.66%
$\frac{1}{7}$	0.1428	14.28%
$\frac{1}{8}$	0.125	12.5%
$\frac{1}{9}$	0.1111	11.11%
$\frac{1}{10}$	0.10	10%
$\frac{1}{11}$	0.0909	9.09%

$$+ \frac{3}{8} = \frac{3 \times 1}{8} = 3 \times 0.125 = 0.375 = 37.5\%$$

$$+ \frac{2}{9} = \frac{2 \times 1}{9} = 2 \times 0.1111 = 22.22\%$$



State

$$+ \frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100}$$

$$+ \text{P\% of a number 'x' = y} \Leftrightarrow \frac{P}{100} \cdot x = y$$

→ ~~Percent~~ Increase & Decrease -

$$\% \text{ change} = \frac{\text{Change}}{\text{original value}} \times 100$$

→ Increase or Decrease :-

If price increased or decreased by same %age.

$$\text{New Price} = \left(1 + \frac{\text{Percent change}}{100}\right) \times \text{original Price}$$

⊕ → increase ; ⊖ → decrease.

→ Simple Interest & Compound Interest :-

$$1) \quad SI = \frac{PRT}{100} ; \text{Amount (A)} = P + SI$$

$$2) \quad A = P \left(1 + \frac{r}{100}\right)^n ; CI = A - P$$

(or)

$$A = P \left(1 + \frac{r}{100}\right)^{nc}$$



State

P = Principal

r = rate in %age.

C = No. of times the interest is compounded per year (or) no. of compounding per year.

n = No. of years.

→ Compounded half yearly → 2 compounding per year.

$$A = P \left(1 + \frac{r}{200}\right)^{2n}$$

$$\hookrightarrow \text{Quarterly} \rightarrow A = P \left(1 + \frac{r}{400}\right)^{4n}$$

→ Monthly → 12 compounding per year.

$$A = P \left(1 + \frac{r}{1200}\right)^{12n}$$

RATIOS :-

→ Proportioning of Ratios :-

15 cookies total ; K : A = 2 : 1 \Rightarrow 2+1=3

$$K = \frac{2}{3} \times 15 \quad \& \quad A = \frac{1}{3} \times 15$$



Date

* Combining Ratios -

Boxes : Paddles

Boxes : Tires

 $2 : 5$ $3 : 4$

B : P

B : T



P : B

B : T

 $5 : 2$ $3 : 4$ $(\times 3)$ $(\times 2)$ $15 : 6$ $6 : 8$ 

P : B : T

 $15 : 6 : 8$

Powers & Roots :-

* Exponents & Bases

 $5 \rightarrow$ exponentBase 2 $1^x = 1$; $0^x = 0$ ($x \neq 0$) ($0^0 = \text{not defined}$) $x^0 = 1$; $x^1 = x$ 

Date

Even & Odd Exponents :-

 \rightarrow Negative no. raised to even power results in a positive outcome

$$(-9)^2 = 81$$

 \rightarrow Negative no. raised to odd power results in a negative outcome.

$$(-3)^3 = -27$$

 \rightarrow An odd exponent preserves sign of base -

$$(-2)^3 = -8 ; (10^5) = 100000$$

 \rightarrow An even exponent always results in a positive outcome

$$(-2)^4 = 16 ; 10^3 = 1000$$

$$x^3 = 8 \Rightarrow x = 2$$

$$x^2 = 4 \Rightarrow x = 2 \text{ or } x = -2$$

* Powers to memorize :-

$2^1 = 2$	$3^1 = 3$	$4^1 = 4$	$5^1 = 5$
$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$
$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$
$2^4 = 16$	$3^4 = 81$	$4^4 = 256$	$5^4 = 625$
$2^5 = 32$			
$2^6 = 64$			
$2^7 = 128$			



Exponential growth \rightarrow

Shute

Positive Bases

base > 1

base < 1

If $x > 1$, then value of x^n increases as value of n \uparrow s
 If $0 < x < 1$, the value of x^n \downarrow s as the value of n \uparrow s
 Moves away from 1 } Moves closer to 1 }

* Similar pattern in observed in negative bases but sign alternates from \oplus & \ominus

\downarrow even power
 \downarrow odd power

EXPONENT LAW:-

1. Product Law -

$$x^a \cdot x^b = x^{a+b} \quad \text{[requires equal bases]}$$

2. Quotient Law -

$$x^a \div x^b = \frac{x^a}{x^b} = x^{a-b} \quad \text{[requires same equal bases]}$$

3. Power of power Law:-
 $(x^a)^b = x^{a \cdot b}$



$$x^{-n} = \frac{1}{x^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

Shute

6. Power of Product Law:-

$$(x^a \cdot y^b)^n = x^{an} \cdot y^{bn}$$

7. Power of Quotient Law:-

$$\left(\frac{x^a}{y^b}\right)^n = \frac{x^{an}}{y^{bn}}$$

8. Combining Base Law:-

$$x^a y^n = (xy)^n ; \frac{x^n}{y^n} = \left(\frac{x}{y}\right)^n$$

THE UNIT DIGIT QUESTIONS:-

1. Look for repeating pattern.

2. Figure out where that pattern will be at desired power.

* The unit digit of any product will be influenced only by unit digits of two factors.

* Square Roots and Squares:-

$$\sqrt{2} = 1.414 \approx 1.4$$

$$\sqrt{3} = 1.732 \approx 1.7$$

$$\sqrt{5} = 2.236 \approx 2.2$$



Shree

$1^2 = 1$	$11^2 = 121$
$2^2 = 4$	$12^2 = 144$
$3^2 = 9$	$13^2 = 169$
$4^2 = 16$	$14^2 = 196$
$5^2 = 25$	$15^2 = 225$
$6^2 = 36$	$16^2 = 256$
$7^2 = 49$	$17^2 = 289$
$8^2 = 64$	$18^2 = 324$
$9^2 = 81$	$19^2 = 361$
$10^2 = 100$	$20^2 = 400$

* If $x > 1$, then $\sqrt{x} < x$

If $0 < x < 1$, then $\sqrt{x} > x$

→ ODD ROOTS:-

1) We can find the odd roots of a negative no.

2) The odd root of negative no. will be negative.

3) The odd root of positive no. will be positive.

→ EVEN ROOTS:-

1) We cannot find even roots of a negative no.

2) Even root of +ve no. will be positive.

< All roots have almost 1 value >



* $\sqrt[n]{x} = x^{1/n}$

Shree

PROPERTIES OF ROOTS:-

1) $\sqrt[n]{x} \cdot \sqrt[n]{y} = \sqrt[n]{xy} \Leftrightarrow \sqrt[n]{xy} = \sqrt[n]{x} \cdot \sqrt[n]{y}$

2) $\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}} \Leftrightarrow \sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$

3) $x^{a/b} = \sqrt[b]{x^a} \Leftrightarrow x^{a/b} = (\sqrt[b]{x})^a$

4) $a^x = a^y \Rightarrow x = y$ (0+0 8 0+1)

Rationalization :- (eliminating roots from denominator)

$$\frac{5}{3+\sqrt{2}} = \frac{5(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})} = \frac{15-5\sqrt{2}}{7}$$

* $(3-\sqrt{2})$ is conjugate of $(3+\sqrt{2})$



ALGEBRA EQUATIONS & INEQUALITIES:-

Date

$$1) (a+b)^2 = a^2 + 2ab + b^2$$

$$2) (a-b)^2 = a^2 - 2ab + b^2$$

$$3) a^2 - b^2 = (a+b)(a-b)$$

$$4) (x+a)(x+b) = x^2 + (a+b)x + ab$$

$$x^2 + mx + p = (x+a)(x+b)$$

$$\begin{array}{cc} \downarrow & \downarrow \\ (a+b) & ab \end{array}$$

* Solutions of equation = Roots of equation

5) Quadratic Equation -

$$ax^2 + bx + c = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{★ (i) } b^2 - 4ac = 0 \quad \text{One Unique soln / both roots same}$$

$$(ii) \quad b^2 - 4ac > 0 \quad \text{Two soln / roots}$$

$$(iii) \quad b^2 - 4ac < 0 \quad \text{No soln / roots do not exist}$$



Numbers of Solutions of System of Linear Equations:-

Date

A system of 2 eqn with 2 variables can have zero solutions, one solution or infinitely many solutions.

1) Infinitely many solutions -

$$\begin{array}{l} 2x + 2y = 9 \\ 6x + 4y = 18 \end{array}$$

$$\Rightarrow \begin{array}{l} 6x + 4y = 18 \\ \underline{(-) 6x + 4y = 18} \\ 0x + 0y = 0 \end{array}$$

< Infinite no. of solution >

2) Zero Solution:-

$$\begin{array}{l} 3x + 2y = 9 \\ 6x + 4y = 11 \end{array}$$

$$\Rightarrow \begin{array}{l} 6x + 4y = 18 \\ \underline{(-) 6x + 4y = 11} \\ 0x + 0y = 7 \end{array}$$

$$0x + 0y = 7$$

< Zero Solution >



Date

One Solution -

$$3x + y = 17$$

$$2x - 2y = 6$$

$$\Rightarrow 6x + 2y = 34$$

$$(+) \quad 2x - 2y = 6$$

$$8x = 40$$

$$x = 5$$

$$\rightarrow 2x - 2y = 6 \Rightarrow 2 \times 5 - 2y = 6$$

$$\Rightarrow y = 2$$

* EQUATIONS WITH SQUARE ROOTS :-

$$* \sqrt{x-2} = 3 \Rightarrow x-2=9 \Rightarrow x=11$$

$$* \sqrt{3x-5} = \sqrt{x+10} \Rightarrow 3x-5 = x+10 \Rightarrow x = 7.5$$

* EXTRA NEOUS ROOTS :-

$$\sqrt{x+1} = -2 \Rightarrow x+1=4$$

$$x=3$$

Now, put $x=3$ in eqn -

$$\sqrt{3+1} = -2 \Rightarrow 2 \neq -2$$

So, $x=3$ is an extraneous root.

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* Extraneous roots can arise when we square numbers as in above case.

$$(\sqrt{k})^2 = k \text{ for } k \geq 0$$

$$\text{At } (\sqrt{9})^2 = 9$$

$$\text{but, } (\sqrt{-9})^2 \neq -9 \text{ \{as } k \neq 0\}}$$

$$\rightarrow \sqrt{5-x} = \sqrt{2x-13}$$

$$5-x = 2x-13 \Rightarrow x=6 \rightarrow \text{Now check for extraneous roots.}$$

$$\sqrt{5-x} = \sqrt{2x-13} \Rightarrow \sqrt{-1} \neq \sqrt{1}$$

\Rightarrow There is no real value.

So, $x=6$ is not a solution of the eqn & thus, given pair of eqn have no solution.

$$\rightarrow \sqrt{x+14} = x+2 \Rightarrow x+14 = (x+2)^2$$

$$x=-5, x=2$$

Now, check for extraneous roots.

$$x=-5 \rightarrow \sqrt{9} = -3 \rightarrow \text{Not possible.}$$

$$x=2 \rightarrow \sqrt{16} = 4$$

So, $x=-5$ is an extraneous root.
 $\therefore x=2$ is the only solution of equation.



Date

* EQUATION WITH SQUARE ROOTS:-

- 1) Eliminate square root by squaring both sides.
- 2) Solve for the variable.
- 3) Check for extraneous roots.

* Equation with n^{th} root:-

- 1) Raise both sides by power n .
- 2) Solve for the variable.
- 3) If n is even - check for extraneous roots.

EQUATIONS WITH ABSOLUTE VALUE:-

$$|x| = x \quad \text{if} \quad x \geq 0 \Rightarrow |4| = 4$$

$$|x| = -x \quad \text{if} \quad x < 0 \Rightarrow |-4| = -(-4) = 4$$

$$\text{In general - } |x| = a \Rightarrow \begin{cases} x = a & ; & x \geq 0 \\ x = -a & ; & x < 0 \end{cases}$$

$$\hookrightarrow |x| = 4 \rightarrow x = 4 \text{ or } x = -4$$

$$\hookrightarrow |x| = 3x - 4 \Rightarrow \begin{aligned} x &= 3x - 4 & \Rightarrow x = 2 \\ x &= -(3x - 4) & \Rightarrow x = -1 \end{aligned}$$



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* When solving equations with Absolute Values, always check for extraneous roots.

So, checking $|x| = 3x - 4$ for extraneous roots -

$$x = 2 \rightarrow |2| = 3 \times 2 - 4 = 2 \quad (\checkmark)$$

$$x = +1 \rightarrow |1| = 3 \times 1 - 4 = -1 \quad (\times)$$

So, $x = 1$ is an extraneous root & thus, $x = 2$ is the only solution.

So, for eqns with Absolute value -

$$1) \text{ Apply rule } |x| = a \Rightarrow \begin{cases} x = a & \rightarrow x \geq 0 \\ x = -a & \rightarrow x < 0 \end{cases}$$

2) Solve eqn for results.

3) Check for extraneous roots.



INEQUALITIES :-

Date

1) Adding or Subtracting to or from both sides does not affect the inequality.

2) Multiplying & dividing both sides by positive numbers doesn't affect the inequality.

3) Multiplying & dividing both sides by negative numbers reverses the inequality.

* Inequality can be added.

* Don't ever subtract, multiply or divide an inequality

$$A < B$$

$$+ \frac{C}{1} < \frac{D}{1}$$

$$A+B < C+D.$$

Inequalities & Absolute value :-

$$1) |x| < 3 \Rightarrow x < 3$$

$$x > -3$$

$$-3 < x < 3$$

$$* |x| < a \Rightarrow -a < x < a \text{ (where } a \text{ is +ve)}$$

$$2) |x| \geq 2 \Rightarrow x \geq 2$$

$$x \leq -2$$



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$$* x > a \Rightarrow x > a \text{ (or) } x < -a$$

(where a is +ve)

Quick :-

$$|x| < 5$$

$$-5 < x < 5$$

$$|x| > 3$$

$$-3 > x > 3$$

Trick :- Put negative of number

to the left of inequality and place inequality sign same as original in place them.

(Trick).

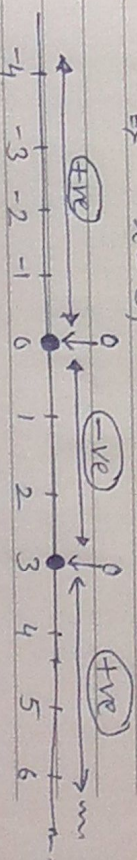
$$x < 3 \text{ (or) } x > 3.$$

QUADRATIC INEQUALITIES :-

$$x^2 - 2x - 3 > 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow x = 3, -1.$$



$$x = 4 \rightarrow x^2 - 2x - 3$$

$$= 5 > 0 \Rightarrow +ve$$

$$x = 2 \rightarrow x^2 - 2x - 3$$

$$= -3 < 0 \Rightarrow -ve$$

$$x = -2 \rightarrow x^2 - 2x - 3$$

$$= 5 > 0 \Rightarrow +ve$$

$$\boxed{-1, 3}$$



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Ex. Solution of equation $x^2 - 2x - 3 < 0$ is -

$$-1 < x < 3. \quad \left\{ \begin{array}{l} \text{In this region the value} \\ x < 0 \end{array} \right\}$$

Steps:-

- 1) Set expression to ~~be~~ equal zero.
- 2) Find solutions & record them on the number line.
The solution will divide the line into 3 regions.
- 3) Test a number from each region.
- 4) Solve the inequality.



Date