QUANTITATIVE COMPARISONS

ALLOWED TO DO THE FOLLOWING

✓ Add or subtract any number from both sides

✓ Multiply or divide both quantitities by any positive number.

- Don't multiply or divide by a variable whose sign is unknown

✓ Cross-multiply, as long as all fractions are positive

- Don't cross-multiply by a variable whose sign is unknown.

✓ Square or square-root both quantities if both are positive.

- Don't ~~square or~~ square-root a variable whose sign is unknown.

COMBINATORICS

FACTORIALS FOR 1 to 6

- ✓ $\underline{|1} = 1$
- ✓ $\underline{|2} = 2$
- ✓ $\underline{|3} = 6$
- ✓ $\underline{|4} = 24$
- ✓ $\underline{|5} = 120$
- ✓ $\underline{|6} = 720$

PERMUTATIONS

✓ $^{n}P_{r} = \dfrac{\underline{|n}}{\underline{|n-r}}$

✓ Use the Fundamental Counting Principle instead.

✓ While solving, always start with the most restrictive stage.

✓ But prefer to use the FCP (Fundamental Counting Principle).

COMBINATIONS

✓ $^{n}C_{r} = \dfrac{\underline{|n}}{\underline{|n-r}\;\underline{|r}}$

$\boxed{^{n}C_{r} = \dfrac{\text{First } r \text{ values of } \underline{|n}}{\underline{|r}}}$

✓ $\boxed{^{n}C_{r} = {}^{n}C_{n-r}}$

✓ If $^{n}C_{x} = {}^{n}C_{y} \Rightarrow x = y$ or $x + y = n$

✓ $^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

✓ $^{n}C_{1} = n$

✓ Combinations are used when the outcomes of one stage are the same as outcomes of other stages.

→ NUMBER OF HANDSHAKES

✓ Always divide the final answer by 2

✓ n = number of people

$$\boxed{\# \text{Handshakes} = \frac{n(n-1)}{2}}$$

→ CIRCULAR PERMUTATIONS

✓ $\underline{|n-1}$ → If considered only in one direction

✓ $\frac{|n-1}{2}$ → If both clockwise & anticlockwise arrangements are considered same

→ NON-COLLINEAR POINTS IN A PLANE

n → no. of points

✓ $\boxed{\# \text{straight lines} = {}^nC_2}$

✓ $\boxed{\# \text{triangles} = {}^nC_3}$

→ WAYS TO CHOOSE 'r' OR MORE OF 'n' OBJECTS

✓ $\boxed{\# \text{ways} = 2^n - r}$

eg. If 10 questions on exam and the student needs to attempt 2 or more

$\therefore$ # ways $= 2^{10} - 2$

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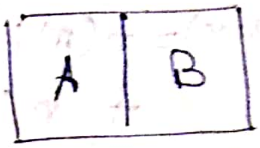
PROBABILITY

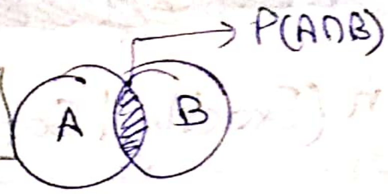
→ MUTUALLY EXCLUSIVE EVENTS (Can't happen at the same time)

✓ $P(A \text{ or } B) = P(A) + P(B)$



→ EVENTS WITH OVERLAP

✓ $\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$



→ INDEPENDENT EVENTS → In cases 'with replacement'

They have no effect on each other's outcomes. (Unrelated)

✓ $P(A \cap B) = P(A) \cdot P(B)$

(CONDITIONAL PROBABILITY)

→ NOT INDEPENDENT EVENTS → In cases 'without replacement'.

✓ $P(A \cap B) = P(B) \cdot P(A|B)$

$= P(A) \cdot P(B|A)$

→ BINOMIAL EXPERIMENT

✓ $\boxed{{}^{n}C_{r}\, p^{r}\, q^{n-r}}$ $(q = 1 - p)$

$n \rightarrow$ # trials

$r \rightarrow$ # successes

$p \rightarrow$ Prob. of success

$q \rightarrow$ Prob. of failure $= 1 - p$

Number Properties

→ Properties of Multiples

✓ e.g. If 2401 is a multiple of 7, then
2401 + 7 = 2408 is a multiple of 7
2401 − 7 = 2394 is a multiple of 7
2408 + 7 = 2415 is a multiple of 7
⋮

✓ eg. If 700 & 49 are multiples of 7, then
700 + 49 = 749 is a multiple of 7
749 + 49 = 798 is a multiple of 7
700 − 49 = 651 is a multiple of 7
⋮

✓ If 52 is a multiple of 13, then
52 × 3 is a multiple of 13
52 × 7 is a multiple of 13
52 × 2 " " " "
⋮

✓ If 24 & 80 are multiples of 8, then
24×80 = 1920 is a multiple of 8
⋮

✓ But, do not divide two multiples of a given number!

→ Prime Numbers → '1' is NOT a prime

✓ factors of a prime are 1 and the number itself

The only even prime → 2

2	11	23	31	41	53
3	13	29	37	43	59
5	17			47	
7	19				

→ TESTING WHETHER A LARGE NUMBER IS PRIME

✓ If a number less than 100 is not divisible by a prime less than 10 i.e. $\{2,3,5,7\}$, then the number has to be prime.

✓ In doing so, always eliminate even numbers because 2 is the only even prime.

⇒ FINDING THE NUMBER OF FACTORS OF LARGE NUMBERS

✓ Find the prime factorisation of N and write it in terms of powers of the primes.

$$8400 = 84 \times 100$$
$$= 21 \times 4 \times 5 \times 2 \times 5 \times 2$$
$$= 3 \times 7 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$$
$$= 2^4 \times 3^1 \times 5^2 \times 7^1$$

✓ Create a list of the exponents

$\{4, 1, 2, 1\}$

✓ Add 1 to every number of the new list

$\{5, 2, 3, 2\}$ → Bcoz every power can range from zero to n, thus $(n+1)$

✓ Find the product of the new list.

$5 \times 2 \times 3 \times 2 = 10 \times 6 = \boxed{60}$ → No. of ways in which (power) blanks can be filled.

→ FOR THE NUMBER OF 'ODD' FACTORS

✓ In the procedure above, ignore the powers of 2.

→ FOR THE NUMBER OF 'EVEN' FACTORS

✓ Subtract the no. of odd factors from the no. of all factors.

✓ No direct method.

* If the powers of all the prime factors of N are even then N is a perfect square for sure

> NUMBER OF FACTORS OF PERFECT SQUARES

✓ Since all powers of all prime factors of perfect squares are even, when we add 1 to each of them in the final step, it will become an odd number.

✓ ∴ Perfect squares always have odd no. of factors.

✓ Perfect squares are the only integers with an odd no. of factors.

> HIGHEST COMMON FACTOR / GREATEST COMMON FACTOR (HCF / GCF)

✓ List the prime factorisation of the integers.

$720 = 2^4 \times 3^2 \times 5^1$

$1200 = 2^4 \times 3^1 \times 5^2$

✓ Find the factors that are common to both.

$\longrightarrow$ 2, 3, 5

✓ Multiply them with lowest powers of each.

∴ $HCF(720, 1200) = 2^4 \times 3^1 \times 5^1 = \boxed{240}$

> LOWEST COMMON MULTIPLE (LCM)

✓ First find the HCF

$24 = 2^3 \times 3$ $\quad 32 = 2^5$

∴ $GCF(24, 32) = 2^3 = 8$

✓ Write the integers as products of GCF and some other number.

$24 = 8 \times 3$ $\quad 32 = 8 \times 4$

✓ LCM is the product of the GCF and all other numbers.

∴ $LCM = 8 \times 3 \times 4 = \boxed{96}$

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→ EVEN & ODD INTEGERS

✓ Evens & odds include both +ve & −ve numbers.

✓ Evens & odds are only Integers.

✓ Zero is an even number.

✓ Any even number can be expressed as $2k$, where k is any integer.

✓ Odd numbers can be expressed as $2k+1$ or $2k-1$. Prime factorization of odd numbers doesn't contain a factor of 2.

* Unless stated in the question, do not assume all variables to be integers alone. They could be real numbers.

→ PRIME FACTORIZATION OF LARGE NUMBERS

✓ Use $a^2 - b^2 = (a+b)(a-b)$

e.g. $1599 = 1600 - 1 = 40^2 - 1^2 = (40+1)(40-1)$
$= (41)(39)$
$= 3 \times 13 \times 41$

e.g. $2491 = 2500 - 9 = 50^2 - 3^2 = (50+3)(50-3)$
$= 53 \times 47$

e.g. $9975 = 10000 - 25 = 100^2 - 5^2 = (100+5)(100-5)$
$= (105)(95)$
$= (5 \times 21)(5 \times 19)$
$= (5 \times 3 \times 7)(5 \times 19)$
$= 3 \times 5^2 \times 7 \times 19$

NUMBER PROPERTIES

* 'Number' means 'real number'

> DIVISIBILITY & PRIMES

* 'Integer' means both positive & negative integers

→ DIVISIBILITY RULES

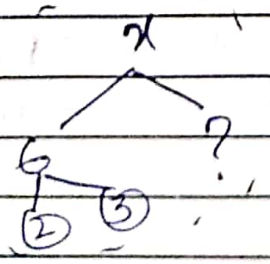
✓ 2 → If the integer is even
✓ 3 → If the sum of digits is divisible by 3
✓ 4 → If the 2-digit number at the end is divisible by 4
✓ 5 → If the integer ends in 0 or 5
✓ 6 → If the Z is divisible by both 2 & 3
✓ 8 → If the 3-digit no. at the end is divisible by 8
✓ 9 → If the sum of digits is divisible by 9

→ THE FACTOR FOUNDATION RULE

✓ If a is divisible by b and b is divisible by c, then a is divisible by c as well.
✓ If d is divisible by 2 different primes e & f, then d is also divisible by e×f.

→ UNKNOWN NUMBERS & DIVISIBILITY

✓ E.g. If some unknown +ve no. 'x' is divisible by 6.



- But it is not mentioned that it is divisible only by 6
- This is incomplete info.
- Consider the following statements

Decide whether it must be / could be / cannot be true

12
6

I x is divisible by 3
II x is even
III x is divisible by 12

I Must be true, becoz 3 is a prime factor of x.
II Must be true, becoz 2 is a prime factor of x
III Could be true, if an extra 2 is also a prime factor of x.

→ TRICK FOR FAST SUBTRACTION

✓ $\boxed{a-b = (a+k) - (b+k)}$

e.g. $56-19 = 57-20 = 37$

→ ROUNDING OF DECIMALS

✓ 0-4 → Round down
5-9 → Round up

✓ Only look at the place value to the immediate right.

e.g. 71432 to the nearest hundreds ≈ 71430
0.413267 to the nearest ten-thousandths ≈ 0.4133

Remainders

$$\boxed{\frac{D}{d} = Q + \frac{R}{d}}$$

D → Dividend
d → Divisor
Q → Quotient
R → Remainder

$$\boxed{0 \leq R < d}$$

→ Properties

✓ If $\frac{D}{d} \to R$, then $\frac{D+kd}{d} \to R$ $(k \in \mathbb{Z})$

e.g. $\frac{1997}{12} \to R5$, then $\frac{1997+12}{12} = \frac{2009}{12} \to R5$

$\frac{1997-12}{12} = \frac{1985}{12} \to R5$

Factors

4 different things →

- Factors of N
- Prime factors of N
- Distinct factors of N
- Distinct prime factors of N

LCM & GCF Formula (For 2 Numbers Only)

$$\boxed{(A)(B) = LCM_{AB} \times GCF_{AB}}$$

D → Dividend
d → Divisor
Q → Quotient
R → Remainder

→ REMAINDERS

$$\boxed{\frac{D}{d} = Q + \frac{R}{d}}$$

$$\boxed{0 \leq R < d}$$

→ RANGE OF POSSIBLE REMAINDERS

✓ When an integer is divided by a positive integer N, the remainders range from 0 to (N-1)

$\cancel{R \in [0, (N-1)]}$

→ ARITHMETIC WITH REMAINDERS

✓ Remainders can be added or subtracted, as long as excess or negative remainders are corrected for.

- $\frac{x}{7} \rightarrow R4$, $\frac{y}{7} \rightarrow R2$

$\Rightarrow \frac{x+y}{7} \rightarrow R4 + R2 = R6$ $(6<7)$

- $\frac{x}{7} \rightarrow R4$, $\frac{z}{7} \rightarrow R5$

$\Rightarrow \frac{x+z}{7} \rightarrow R4 + R5 = R9$ (But $9>7$)

$= R(9-7)$

$= R2$

- $\frac{x}{7} \rightarrow R4$, $\frac{z}{7} \rightarrow R5$

$\Rightarrow \frac{x-z}{7} \rightarrow R4 - R5 = R(-1)$ (But $(-1)<0$)

$= R(-1+7)$

$= R6$

✓ Remainders can be multiplied, as long as excess remainders are corrected for.

- $\frac{x}{7} \to R4$, $\frac{z}{7} \to 5$

$\Rightarrow \frac{xy}{7} \to (R4)(R5) = R20$ (But $20 > 7$)

$= R(7+7+6)$

$= R6$ (Taking out two 7's)

⇒ DIVIDING N BY 5

✓ $\boxed{\frac{N}{5} = \frac{2N}{10}} \to$ Faster

⇒ SQARING MULTIPLES OF 5

✓ Two digits → a & 5 → a5

$5^2 = 25$ Ans → $\boxed{(a)(a+1)\ 25}$

$(a)(a+1) = ?$

e.g. $75^2 = ?$

$(7)(7+1) = 7 \times 8 = 56 \Rightarrow \boxed{75^2 = 5625}$

→ ODDS & EVENS

→ ARITHMETIC RULES

Add or Subtract 'likes' ⇒ Even	✓	Odd ± Odd = Even
		Even ± Even = Even
Add or subtract 'unlikes' ⇒ Odd	✓	Odd ± Even = Odd

Only if there is at least one Even factor do we get an Even product.	✓	(Odd) (Even) = Even	If the product is odd, then each number is Odd for sure.
		(Even) (Even) = Even	
		(Odd) (Odd) = Odd	

→ SUM OF TWO PRIMES

✓ 2 is the only even prime. So if the sum of two primes is odd, ~~one~~ one of them must be 2.

→ ARITHMETIC RULES FOR DIVISION

✓ $\frac{\text{Even}}{\text{Even}}$ → Even or Odd or Not an Integer

✓ $\frac{\text{Odd}}{\text{Odd}}$ → Odd or Not an Integer

✓ $\frac{\text{Even}}{\text{Odd}}$ → Even or Not an Integer

✓ $\frac{\text{Odd}}{\text{Even}}$ → Not an Integer

⟩ ROOTS

→ STRATEGIES

✓ If the $\sqrt{\ }$ sign is explicitly mentioned in the question, consider positive roots only.
If the question contains x^2 or leads to x^2 or requires you to calculate $\sqrt{x^2}$, then consider both positive & negative roots.

→ IMPORTANT ROOTS

$\sqrt{2} \simeq 1.4$ $\qquad$ $\sqrt{3} \simeq 1.7$ $\qquad$ $\sqrt{5} \simeq 2.2$

$\boxed{* \text{ If } \underbrace{k \in (0,1)}_{\text{Proper Fraction}}, \text{ then } \sqrt{k} > k.}$

→ PROPERTIES

✓ If $k \in [1, \infty)$, and if $n > m$, then

$1 < k < \sqrt[n]{k} < \sqrt[m]{k}$. $\qquad$ e.g. $1 < 3 < \sqrt[3]{3} < \sqrt{3}$

✓ If $k \in (0,1)$, then $0 < k < \sqrt[m]{k} < \sqrt[n]{k} < 1$

e.g. $0 < \frac{2}{5} < \sqrt{\frac{2}{5}} < \sqrt[3]{\frac{2}{5}} < \sqrt[4]{\frac{2}{5}} < \dots < 1$

✓ $k^{p/q} = k^{p(\frac{1}{q})} = k^{(\frac{1}{q})p}$.

e.g. $2^{3/5} = (2^3)^{1/5} = \sqrt[5]{2^3}$

or $2^{3/5} = (2^{1/5})^3 = (\sqrt[5]{2})^3$

EXPONENTS

→ THE EVEN EXPONENT IS DANGEROUS

✓ Because it hides the sign of the base

✓ So be careful.

→ A BASE OF 0, 1, -1

✓ $0^{(\text{Anything})} = 0$

✓ $1^{(\text{Anything})} = 1$

✓ $(-1)^{\text{Even}} = 1$

$(-1)^{\text{odd}} = -1$

e.g. If $x^6 = x^7 = x^{15}$, $x = ?$

Plug 0 or 1 or (−1)

$(-1)^6 = 1$

$(-1)^7 = -1$ $\Rightarrow (-1)^6 \neq (-1)^7$

$\therefore \boxed{x = 0 \text{ or } 1}$

→ A FRACTIONAL BASE

✓ As the exponent of a fraction b/w 0 and 1 increases, the value of the expression decreases.

e.g. $\left(\frac{3}{4}\right)^1 > \left(\frac{3}{4}\right)^2 > \left(\frac{3}{4}\right)^3$

$\Rightarrow (0.75)^1 > (0.75)^2 > (0.75)^3$

Powers Of 2

$2^1 = 2$	$2^6 = 64$
$2^2 = 4$	$2^7 = 128$
$2^3 = 8$	$2^8 = 256$
$2^4 = 16$	$2^9 = 512$
$2^5 = 32$	$2^{10} = 1024$

Powers Of 3

$3^1 = 3$	$3^3 = 27$
$3^2 = 9$	$3^4 = 81$

Powers Of 4

$4^1 = 4$	$4^3 = 64$
$4^2 = 16$	$4^4 = 256$

Powers Of 5

$5^1 = 5$	$5^3 = 125$
$5^2 = 25$	$5^4 = 625$

Cubes Of 6–9

$6^3 = 216$	$8^3 = 512$
$7^3 = 343$	$9^3 = 729$

Surds

✓ If $a \pm \sqrt{b} = c \pm \sqrt{d} \Rightarrow a = c$ & $b = d$

✓ If $a + \sqrt{b} = c + \sqrt{d} \Rightarrow a - \sqrt{b} = c - \sqrt{d}$

CONSECUTIVE INTEGERS

→ EVENLY SPACED SEQUENCES

✓ $\boxed{Md = \bar{X} = \frac{a+l}{2}}$

✓ $\boxed{Md = \bar{X} = \frac{\Sigma X_i}{n}}$

→ COUNTING INTEGERS

For Consecutive integers

✓ $a \rightarrow$ First term, $l \rightarrow$ last term

$\therefore \boxed{\#\text{ integers} = l - a + 1}$

✓ For Consecutive Multiples

$\boxed{\#\text{ integers} = \left(\frac{l-a}{d}\right) + 1}$ $d \rightarrow$ Common difference

→ PROPERTIES / FACTS.

✓ A set of n consecutive integers will always contain one number divisible by n.

✓ If n is odd, the sum of a set of n consecutive integers will always be divisible by n.

✓ In a set of 3 consecutive integers → 2E & 1 odd or 1 odd & 2 E

In a set of 4 consecutive integers → 2E & 2 odds

INTEGER STRATEGIES

✓ Don't assume variables to be integers until explicitly mentioned so or unless they are defined as 'even', 'odd' or 'prime'.

✓ Don't forget about zero & negatives

✓ Primes are always positive

✓ If talked about remainders, then all numbers involved are positive integers.

✓ Zero is neither positive nor negative.

✓ Zero is even.

✓ 'One' is not a prime.

✓ 2 is the only even prime.

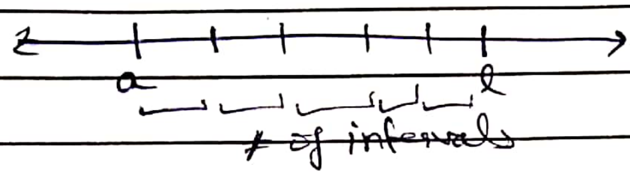
✓ For questions on odd & evens, test all four cases for the variables (both odd, both even, even & odd, odd & even) by substituting '1' for odd and '2' for even.

✓ For questions on consecutive integers, most of these appear in variable form, so simplify the algebraic expression and recognize that they are consecutive.

factorize

NUMBER LINES

→ LENGTH OF AN INTERVAL



$$\boxed{\text{length of an interval} = \frac{l - a}{\#\text{ of intervals}}}$$

$$\boxed{\#\text{ of Intervals} = 1 - (\#\text{ of tick marks})}$$

UNITS' DIGITS

2 → 2, 4, 8, 6	8 → 8, 4, 2, 6
3 → 3, 9, 7, 1	7 → 7, 9, 3, 1
4 → 4, 6, 4, 6	9 → 9, 1, 9, 1
5 → 5, 5, 5, 5	
6 → 6, 6, 6, 6	

ABSOLUTE VALUE

$$|x| = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$$

Sum of the first n odd nos. = n^2

Sum of the first n even nos. = $n(n+1)$

ALGEBRA

RULES FOR INEQUALITIES

→ ADDITION

✓ Inequalities with signs in the same direction can be added.

$$\begin{array}{rl} & a > b \\ (+) & c > d \\ \hline & (a+c) > (b+d) \end{array} \qquad \begin{array}{rl} & 5 > 2 \\ (+) & 8 > 3 \\ \hline & 13 > 5 \end{array}$$

→ SUBTRACTION

✓ NEVER subtract inequalities if signs are in the same direction because no conclusions can be drawn

$$\begin{array}{rl} & 15 > 8 \\ (-) & 10 >_{(-)} 2 \\ \hline & 5 < 6 \end{array} \qquad \begin{array}{rl} & 15 > 8 \\ (-) & 10 >_{(-)} 3 \\ \hline & 5 = 5 \end{array} \qquad \begin{array}{rl} & 15 > 8 \\ (-) & 10 >_{(-)} 4 \\ \hline & 5 > 4 \end{array}$$

✓ But, you can subtract inequalities if their signs are in the opposite direction. And the sign of the result follows that of the first inequation.

$$\begin{array}{rl} & a > b \\ (-) & d <_{(-)} c \\ \hline & (a-d) > (b-c) \end{array} \qquad \begin{array}{rl} & 20 > 15 \\ (-) & 10 <_{(-)} 12 \\ \hline & 10 > 3 \end{array} \qquad \begin{array}{rl} & 12 > 10 \\ (-) & 15 <_{(-)} 20 \\ \hline & -3 > -10 \end{array}$$

→ MULTIPLICATION

✓ Works only if the numbers are positive i.e R^+

$$\begin{array}{r} 5 > 3 \\ \times\ 20 > 19 \\ \hline 100 > 57 \end{array} \qquad \begin{array}{r} 0.5 > 0.3 \\ \times\ 0.1 > 0.05 \\ \hline 0.05 > 0.015 \end{array}$$

BUT → $$\begin{array}{r} (-10) < 2 \\ \times\ (-8) < 3 \\ \hline 80 > 6 \end{array}$$

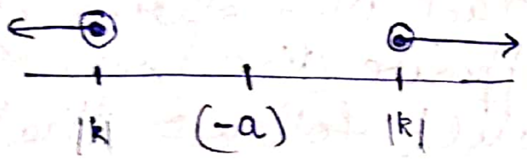
→ DIVISION

✓ Don't do it !!!

> ABSOLUTE VALUES & INEQUALITIES

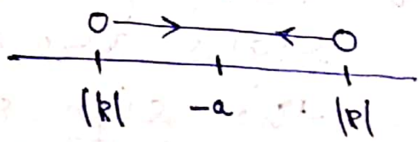
✓ $|x+a|$ → Distance of x from $(-a)$.

If $|x+a| > k$



$\Rightarrow$ $x \in \,]-k, k[$

If $|x+a| < k$



$\Rightarrow$ $x \in (-k, k)$

✓ Sign after $|x+a|$

$> \Longrightarrow x \in \,)\_,\_($ $\qquad$ $< \Longrightarrow x \in (\_,\_)$

$\geqslant \Longrightarrow x \in \,]\_,\_[$ $\qquad$ $\leqslant \Longrightarrow x \in [\_,\_]$

✓ $|x+a| \ \boxed{}\ k$ anyone of these signs

Then solve for $x+a=k$ & $x+a=-k$

And use the rules above to determine the range for x.

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⇒ FORMULAS — PROPORTIONAL REASONING

→ ONLY ONE QUANTITY CHANGES

✓ $v^2 = 2ad$

e.g. If v doubles and a stays the same, by what factor does d change?

$v^2 = 2 . a . d.$

$(1)^2 = 1 . 1 . 1$ ⟶ Start with everything as 1

$(2)^2 = 2 . 1 . d$ ⟶ Now change as mentioned

$\Rightarrow \frac{4}{2} = d \Rightarrow \boxed{d = 2}$

→ MORE THAN ONE QUANTITY CHANGES

e.g. If v triples and a doubles, by what factor does d change?

$v^2 = 2ad$

$1^2 = 1 . 1 . 1$

$(3)^2 = 2 . 2 . d$

$\Rightarrow \boxed{d = \frac{9}{4}}$

ALGEBRA

INEQUALITIES

- ✓ When multiplying or dividing by a (-ve) number, switch the inequality sign.

e.g. $-2x > 10$

$\Rightarrow 2x < -10$

- ✓ Do not multiply or divide an inequality with a variable because you don't know the sign of the 'hidden number'.
- ~~✓ Change sign when cross-multiplying.~~ $\cancel{\frac{a}{x} > \frac{b}{y} \Rightarrow ay < bx}$

ABSOLUTE VALUES

- ✓ Isolate the modulus terms on the LHS of the equality / inequality.

$$|x| = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$$

- ✓ The expression inside the modulus can also be compound.

e.g. $|2x+3| - 10 = 20$

$\Rightarrow |2x+3| = 30$

$(2x+3) = 30$ $\qquad$ $-(2x+3) = 30$

If E stands for any algebraic expression then

$|E| = k$

$E = k$ or $E = -k$

Finally, check if all solutions work by plugging them into the original eqn. Some solns. might be extraneous.

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POLYNOMIAL

✓ An expression with more than one term involving only one variable.

e.g. ✓ $x^2 + 2x + 2$ ✓ ✗ $a^2+b^2+c^2$ ✗

DEGREE

✓ Highest power of an expression

→ LINEAR EXPRESSION

✓ Degree = 1

e.g. $17x$ → Linear monomial
$3x-5$ → Linear binomial

→ QUADRATIC EXPRESSION

✓ Degree = 2

e.g. $14x^2$ → Quadratic monomial
x^2-4, $2x^2+8x$ → Quadratic binomials

EVEN POWERS IN QUADRATIC BINOMIALS ~~WITH CONSTANT ONLY~~

✓ In such expressions where the variable has an even power and there ~~other term~~ is just one other term, think of factoring it as $\boxed{a^2-b^2 = (a+b)(a-b)}$.

e.g. $x^6 - 16 = (x^3-4)(x^3+4)$

$x^8 - 9y^2 = (x^4-3y)(x^4+3y)$

$x^4 - 81 = (x^2+9)(x^2-9) = (x^2+9)(x-3)(x+3)$

FACTORING DECIMALS

✓ Use $a^2-b^2 = (a+b)(a-b)$

e.g. $0.9991 = 1 - 0.0009 = 1^2 - (0.03)^2 = (1-0.03)(1+0.03)$
$= (0.97)(1.03)$

COMPOUND INEQUALITIES

✓ Apply each operation to all terms of a compound inequality

e.g. $1 > 1 - ab > 0$

$\Rightarrow 1 - 1 > 1 - ab - 1 > 0 - 1$

$\Rightarrow 0 > -ab > -1$

$\Rightarrow 0 < ab < 1$

OPTIMIZATION

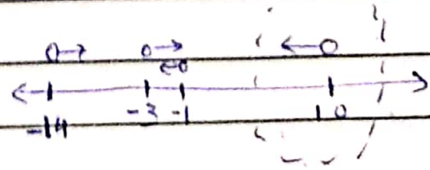
✓ Create a table with min & max. values of each variable and then find the optimum for each combination

✓

+ LT (+ve no.)	=	− GT (−ve no.)
− LT (−ve no.)	=	+ GT (+ve no.)
+ ~~G~~LT (−ve)	=	− GT (+ve)

e.g. If $m \in [-4, 7]$, then integer value of max $(m - n) = ?$
$n \in (-3, 10)$

m	n	m − n
−4	GT(−3)	−4 − GT(−3) = −4 + LT3 = LT(−1)
7	GT(−3)	7 − GT(−3) = 7 + LT3 = LT10
−4	LT10	−4 − LT10 = −4 + GT(−10) = GT(−14)
7	LT10	7 − LT10 = 7 + GT(−10) = GT(−3)



Biggest integer LT10 is $\boxed{9}$.

$\Rightarrow$ SYSTEM OF EQUATIONS

✓ If a system of equations is given and the value of an 'expression' is asked for (rather than that of variable(s)), then don't solve for the variables. Manipulate the equations to get the expression directly.

e.g. $5x+2y=55$, $2x-y=19$, $x+y=?$

$$\begin{aligned} 5x+2y &= 55 \\ (-)2x \overset{-}{(+)} y &= (-)19 \\ \hline 3x+3y &= 36 \end{aligned}$$

$$\Rightarrow \boxed{x+y=12}$$

Overlap (Infinite Solns) ← ✓ If Eqn.(1) = k. Eqn(2), there are infinite solutions because the two equations overlap each other.

Parallel Lines (No Soln). ← ✓ If LHS of Eqn(1) = k LHS of Eqn(2) but RHS of Eqn(1) ≠ k RHS of Eqn(2) then there is no solution because the same thing can not be equal to two different things.

SEQUENCES & ARITHMETIC PROGRESSIONS

→ n^{th} TERM OF AN A.P.

$$\boxed{a_n = a + (n-1)d}$$

$a \rightarrow$ 1st Term
$d \rightarrow$ Common difference

→ SUM OF FIRST n TERMS OF AN AP

$$\boxed{S = \frac{n}{2}\left[2a + (n-1)d\right]}$$

$$\boxed{S = \frac{n}{2}(a + l)}$$

$l \rightarrow$ Last Term

GEOMETRIC PROGRESSIONS

→ n^{th} TERM OF A GP

$$\boxed{a_n = a r^{n-1}}$$

→ SUM OF FIRST n TERMS OF A GP

$$\boxed{S_n = a\frac{(r^n - 1)}{r - 1}}$$

FDP

(Fractions, Decimals, Percents)

Proper Fractions & Integers

PF → Proper fraction, Z → Integers

→ Multiplication

$(Z) \times (PF) < Z$ → $3 \times \frac{1}{3} = 1 < 3$

$(PF_1) \times (PF_2) < (PF_1)$ → $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8} < \frac{1}{2}$

→ Division

$\frac{(Z)}{(PF)} > Z$ → $6 \div \frac{3}{4} = 8 > 6$

$\left(\frac{PF_1}{PF_2}\right) > (PF_1)$ → $\frac{1}{4} \div \frac{5}{6} = \frac{3}{10} > \frac{1}{4}$

Comparing Fractions

✓ Always cross-multiply to compare fractions.

$$\frac{a}{b} \quad \frac{c}{d}$$

Smart Numbers

✓ Take the LCM of denominators and assign that value to the smaller whole.

✓ Do this only if no other part of a whole is mentioned in the equations, i.e. only fractions are mentioned.

$\frac{3}{8}x + \frac{5}{12}x + r_1 = x$

$\frac{1}{4}x + \frac{1}{2}x + r_2 = 2x$

LCM of 8, 12, 4, 2 = 24

So, $x = 24$ → (Smart Number)

$\frac{1}{3}x + \frac{1}{5}x + \frac{3}{10}x + 2 = x$

Don't use smart numbers because of this.

FDP (Fractions, Decimals, and Percentages)

$\boxed{\text{* 'Fractions' means both positive \& negative fractions.}}$

→ Interest Formulas

→ Simple Interest

$$\boxed{I = P \cdot r \cdot t}$$

$I \rightarrow$ Interest amount
$P \rightarrow$ Principal
$r \rightarrow$ Rate in decimals
$t \rightarrow$ Time in terms of no. of yrs.

→ Compound Interest

Look at answer choices and decide whether to use CI formula or to estimate using SI formula

Can be estimated using SI formula. CI will always be above SI for $n>1$

$\Rightarrow \boxed{CI > SI,\ n>1}$

$$\boxed{A = P\left(1 + \frac{r}{n}\right)^{nt}}$$

$n \rightarrow$ No. of times compounded per year.
$A \rightarrow$ Total amount or loan balance

$\boxed{\text{As } n\uparrow,\ A\uparrow}$

→ Proportion

✓ Fraction$_1$ = Fraction$_2$ → Proportion

$$\frac{a}{b} = \frac{c}{d}$$

→ Percent Change

✓ $\boxed{m = \frac{New}{Old}}$

$m \rightarrow$ multiplier (could be something like 1.3 or 0.70, so convert it to %)

$$\boxed{\%\Delta = (m-1)\,100\,\%}$$

→ Sequential Percent Changes

✓ $\boxed{\text{Overall \% Change} = m_1 \cdot m_2 \cdot m_3 \cdots m_n}$

→ Ratios

✓ The simplest form of a fraction → $\frac{a \cdot n}{b \cdot n}$ → useful to write in this manner

→ Unitary Method

✓ किस unit ~~में~~ में answer चाहिए उसे unknown रखो

Km	miles
8	5
x	1

DECIMAL VALUES

* For values of other numerators, just multiply the memorized value by the numerator.

$\frac{1}{2} = 0.5$

$\frac{1}{3} = 0.\overline{3}$

$\frac{1}{4} = 0.25 \longrightarrow \frac{1}{25} = 0.04$

$\frac{1}{5} = 0.2$

$\frac{1}{6} = 0.1\overline{6}$

$\frac{1}{7} = 0.143$

$\frac{1}{8} = 0.125$

$\frac{1}{9} = 0.\overline{1}$

e.g. $\frac{5}{6} = 5 \times (\frac{1}{6}) = 5(0.1\overline{6})$

$= 0.8\overline{3}$

REPEATING DECIMAL TO FRACTION

✓ If a, b, c ... are digits, then

$0.\overline{abc} = \frac{abc}{999}$

e.g. $\boxed{0.\overline{12} = \frac{12}{99}}$ ✓

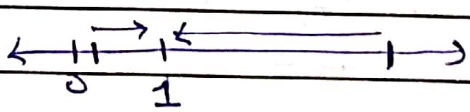
FRACTION PROPERTIES

✓ Reciprocal of an integer (Z)

- If $|Z| > 1$, then $\frac{1}{|Z|} \in (0,1)$ & vice versa

✓ Adding the same numbers to the numerator & denominator

- $\frac{a+c}{b+c}$ makes $\frac{a}{b}$ closer to 1 on the number line



✓ Adding different numbers to the numerator & denominator

- $\frac{a+c}{b+d}$ makes $\frac{a}{b}$ closer to $\frac{c}{d}$ on the number line.

ALGEBRAIC TRANSLATIONS

✓ E is 10% more than F $\longrightarrow$ $\boxed{E = (1.1)F}$

✓ G is 30% less than H $\longrightarrow$ $\boxed{G = (0.7)H}$

SPECIAL WORD PROBLEM

✓ If 'n' is a ratio and if there are 'n' as many x as y, then

$$\boxed{\frac{x}{y} = n}$$

e.g. $\frac{2}{3}$ as many juniors as seniors

$\Rightarrow \frac{J}{S} = \frac{2}{3} = 0.6\overline{6}$

40% as many men as women

$$\frac{M}{W} = \frac{40}{100} = 0.4$$

PERCENT CHANGE FORMULA

✓ Use this when asked 'What percent More?'

$$\boxed{\%\Delta = \left(\frac{\text{Final} - \text{Initial}}{\text{Initial}}\right) \times 100}$$

MIXED RECURRING DECIMAL TO FRACTION

✓ x is a number with 'a' digits and y is a number with 'b' digits

$$\boxed{0.x\overline{y} = \frac{x\overline{y} - x}{b\,9\text{'s}\ a\,0\text{'s}}}$$

e.g. $7.5\overline{36} = 7 + \frac{536 - 5}{990} = 7 + \frac{531}{990}$

PROPERTIES

✓ $\frac{a+k}{b+k} < \frac{a+2k}{b+2k} < \frac{a+3k}{b+3k} \ldots$

e.g. $\frac{4}{5} < \frac{5}{6} < \frac{6}{7} \Leftrightarrow \frac{4}{5} < \frac{4+1}{5+1} < \frac{4+2}{5+2}$

✓ $\frac{a+c}{b+d} < \frac{a+2c}{b+2d} < \frac{a+3c}{b+3d} \ldots$

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> SOLVING FRACTIONS WITH DECIMALS LESS THAN 1

✓ Use $a^2 - b^2 = (a+b)(a-b)$

e.g. $\dfrac{0.999951}{0.993} = \dfrac{1-0.000049}{1-0.007} = \dfrac{(1+0.007)\cancel{(1-0.007)}}{\cancel{(1-0.007)}}$

$= \boxed{1.007}$

e.g. $\left(\dfrac{0.999856}{0.988} - 1\right) = \left(\dfrac{1-0.000144}{1-0.012} - 1\right) = \left[\dfrac{(1+0.012)\cancel{(1-0.012)}}{\cancel{(1-0.012)}} - 1\right]$

$= 1 + 0.012 - 1$

$= \boxed{0.012}$

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Q.C.

FDP

→ FRACTIONS WITH EXPONENTS

✓ Plug in 0 & 1 in place of unknown exponent

		Ⓐ		Ⓑ
		$\frac{1}{2^x}$		2^x
$\boxed{D}$	$x=0$	$\frac{1}{2^0} = \frac{1}{1} = 1$	$=$	$2^0 = 1$
	$x=1$	$\frac{1}{2^1} = \frac{1}{2}$	$<$	$2^1 = 2$

→ FRACTIONS & RECIPROCALS

✓ If one qty. is arithmetic with fractions and the other is the reciprocal of the same arithmetic, don't calculate.

	Ⓐ		Ⓑ
	$\underbrace{\underbrace{\frac{1}{3} + \frac{1}{4}}_{0.5^+} + \underbrace{\frac{7}{12}}_{0.5^+}}_{1^+}$		$\frac{1}{\frac{1}{3} + \frac{1}{4} + \frac{7}{12}}$
$\boxed{A}$			$\frac{1}{1^+}$
	1^+	$>$	$\Downarrow$ 1^-

⇒ PRICE CHANGES & PERCENTS

✓ Pick numbers for fast calculations.

A price increased $x\%$ from 2003 to 2004. Then it decreased by $x\%$ from 2004 to 2005.

$x > 0$, $x \to Z^+$.

A	B
Price of 2004 − Price of 2003.	Price of 2004 − Price of 2005

Let 2003 price = 100

Let x = 10%

⇒ 2004 price = 100 + 10 = 110

⇒ 2005 price = $110 - \left(\frac{10}{100} \times 110\right) = 110 - 11 = 99$

$\boxed{B}$

A		B
110 − 100		110 − 99
= 10	<	= 11

COORDINATE GEOMETRY

SLOPE

✓ Points → (x_1, y_1) & (x_2, y_2)

$$\boxed{\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}} = \frac{\Delta y}{\Delta x}$$

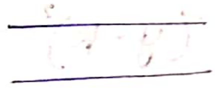
✓ Visually, it means a run of Δx units & a rise of Δy units

or

a run of $(k.\Delta x)$ units and a rise of $(k.\Delta y)$ units.

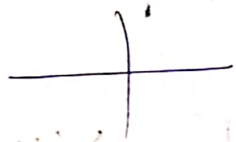
→ PARALLEL LINES

✓ $\boxed{m_1 = m_2}$



→ PERPENDICULAR LINES

✓ $\boxed{m_1 = \left(\frac{-1}{m_2}\right)}$



INTERCEPT

✓ Points at which the line crosses the axes.

- For the x-intercept, put $y=0$ in the eqn.
- For the y-intercept, put $x=0$ in the eqn.

SLOPE-INTERCEPT FORM OF THE LINE

✓ $\boxed{y = mx + c}$

$m \rightarrow$ slope
$c \rightarrow$ y-intercept

SLOPE-POINT FORM OF THE LINE

✓ $\boxed{y - y_1 = m(x - x_1)}$

$m \rightarrow$ Slope
Point $\rightarrow (x_1, y_1)$

Distance Between 2 Points

/ Always draw it on the Cartesian plane to determine the sides of the Δ because they are <u>mostly</u> Pythagorean triplets on the GRE.

✓ If not, $\boxed{d = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}}$

Equation Of A Circle

→ With Centre At (h, k)

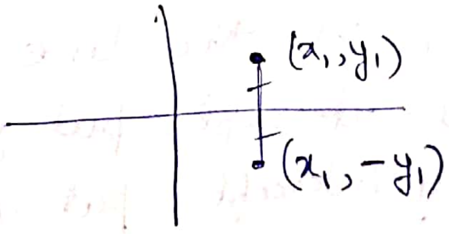
$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

→ With Centre At The Origin

$$\boxed{x^2 + y^2 = r^2}$$

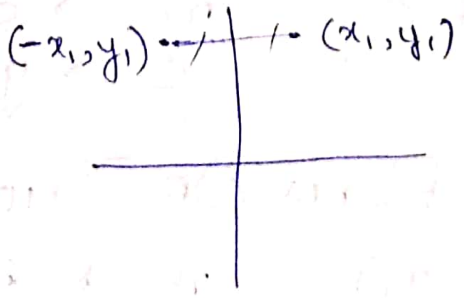
Reflections In The X-Y Plane

→ Reflections Along The X-Axis

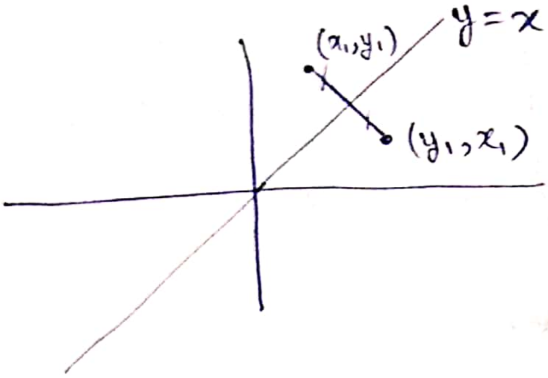


The points would be given and the question would want you to recognize the pattern

→ Reflections Along The Y-Axis

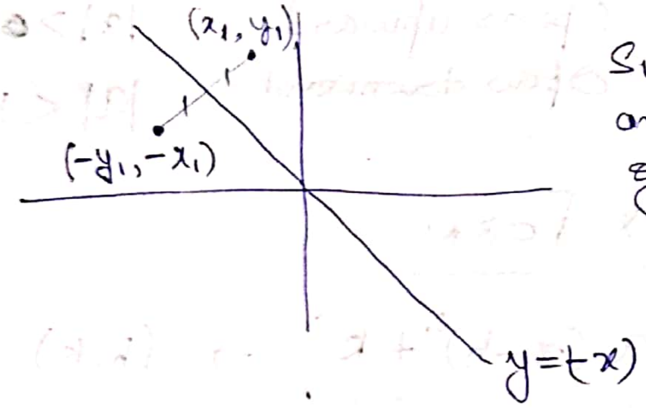


→ REFLECTIONS ALONG THE LINE $y = x$



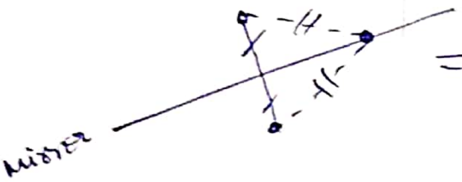
Swap the x- & y- coordinates.

→ REFLECTIONS ALONG THE LINE $y = (-x)$



Swap the coordinates and take negative of both.

→ REFLECTIONS ALONG ANY LINE IN GENERAL



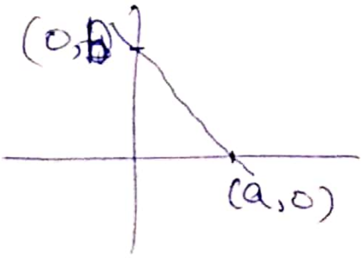
$\Rightarrow$ Any point on the mirror line would be equidistant from the two points.

→ INTERCEPT FORM OF THE LINE

✓ $\boxed{\dfrac{x}{a} + \dfrac{y}{b} = 1}$

$a \rightarrow$ x-intercept

$b \rightarrow$ y-intercept



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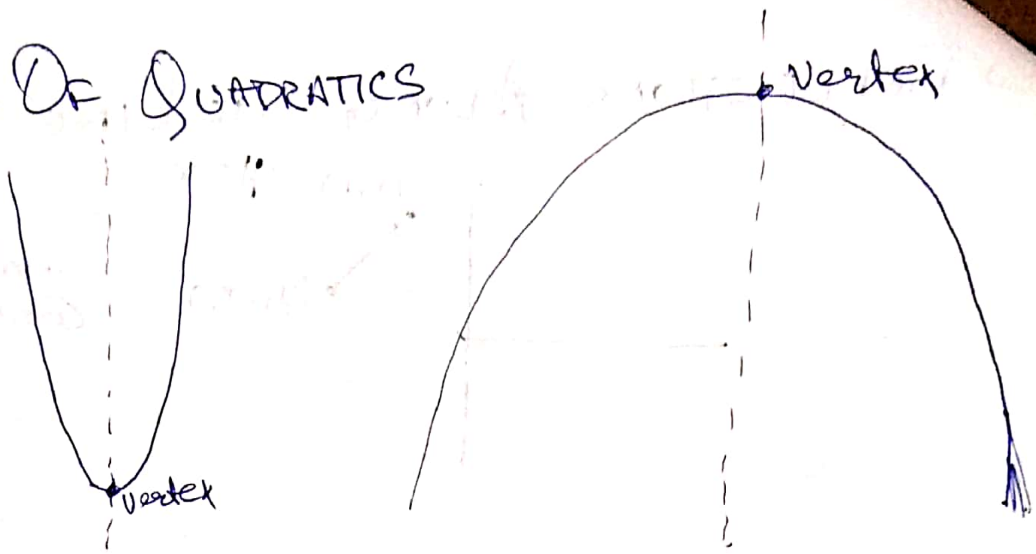
→ Graphs Of Quadratics



$$\boxed{y = ax^2 + bx + c}$$

$a > 0 \rightarrow$ Opens upward

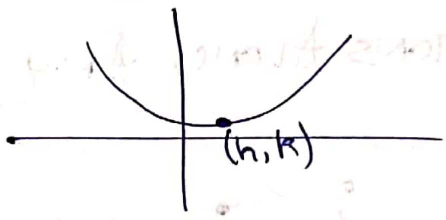
$a < 0 \rightarrow$ Opens downward

$|a| > 1 \rightarrow$ Skinny

$|a| < 1 \rightarrow$ Wide.

→ Vertex Form

$$\boxed{y = a(x-h)^2 + k} \rightarrow (h, k) \rightarrow \text{Vertex}$$



GEOMETRY

→ GEOMETRY STRATEGIES

✓ Extend lines in or add lines ~~in~~ to diagrams.

✓ In some problems, assign variables to either lengths or to angles.

✓ In complicated figures, try to look at the big picture. e.g. Instead of looking at smaller triangles, try to search for one big triangle that encompasses them all.

✓ Whenever more than 2 Δ's are involved (especially with parallel lines), look for similar Δ's.

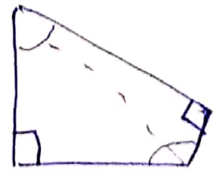
✓ If a right ∠ is present, immediately think of the Pythagorean Theorem.

✓ Know the Pythagorean Triplets and recognize their multiples.

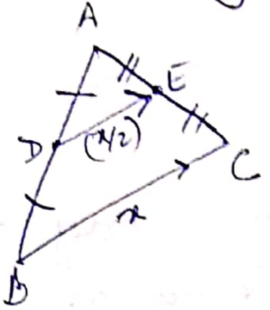
✓ The two special right Δ's can be specified with very little info, but give a great deal of info.

✓ Remember the 2 cases where right ∠'s can be specified bcoz of circular properties (∠ inscribed by the diameter and tangency).

✓ A single pair of ~~Opposite~~ right angles in a quadrilateral do not necessarily produce a regular shape.



✓ The line joining the mid-pts. of any 2 sides of a Δ is parallel to the third side and is half of it.



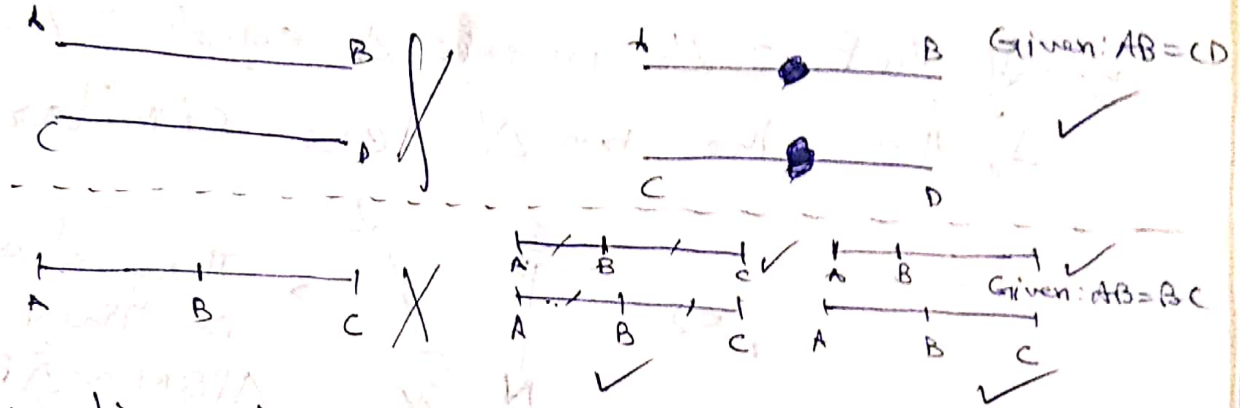
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{BC}{DE} = \frac{2}{1}$$

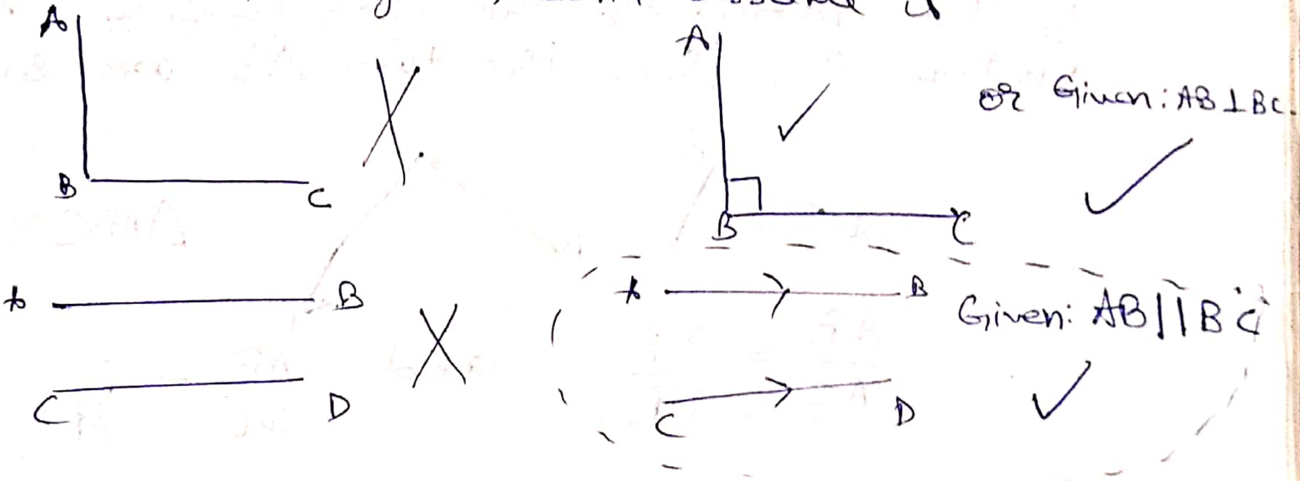
✓ Any line parallel to one side of a Δ divides the other two sides proportionally.

→ WHAT YOU SHOULDN'T ASSUME ON THE GRE

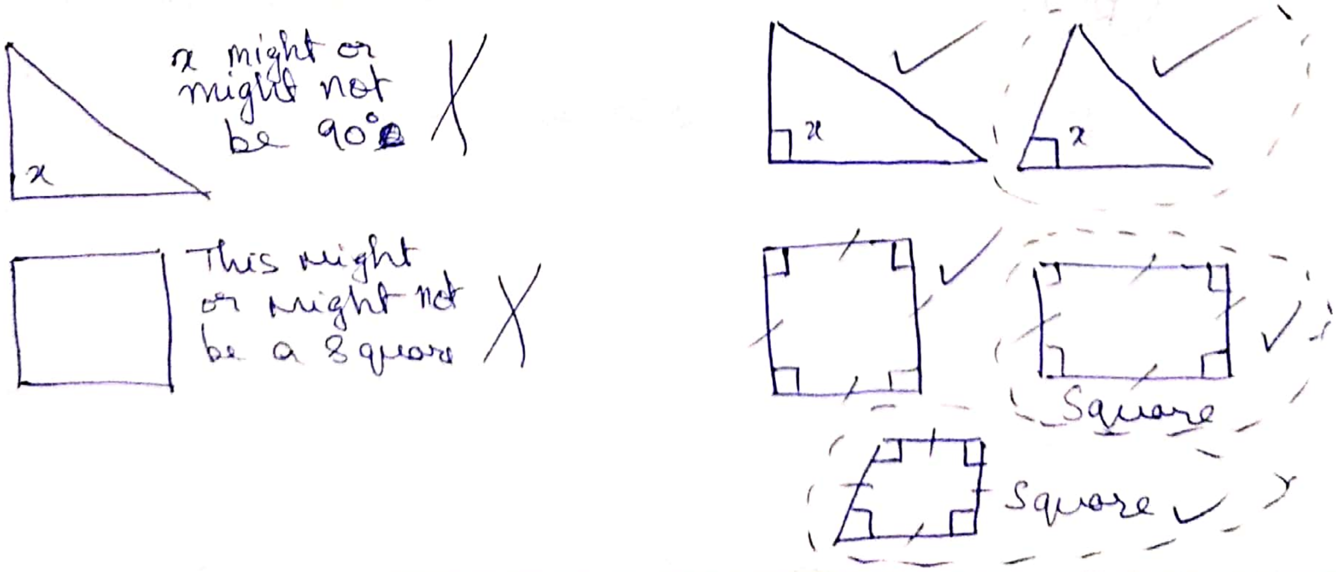
✓ If two lengths look the same but no indication is given, don't assume it.



✓ If two lines look parallel or perpendicular but no indication is given, don't assume it



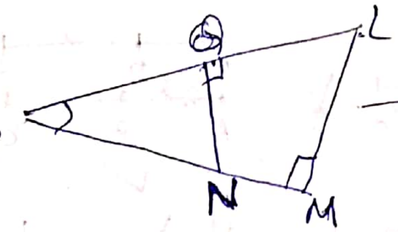
✓ Even if diagrams are drawn to scale, don't assume angles to be equal to what they appear to be unless indicated.



> SIMILAR TRIANGLES (PROPORTIONALITY)

✓ Same shapes (maybe flipped) but different sizes.

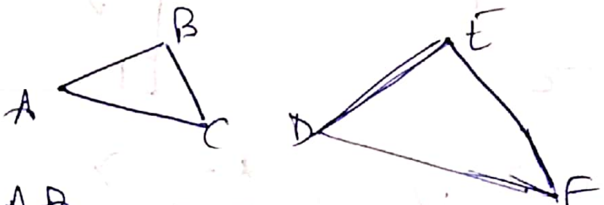
✓ If just 2 ∠'s in one Δ equal 2 ∠'s in another Δ, then the two Δ's are similar.



They share ∠P & ∠Q = ∠M so they are similar

$\Delta PQN \sim \Delta PML$

✓ If the ratio of any 2 sides of one Δ is equal to the ratio of the corresponding sides in another Δ, then the two Δ's are similar.



$\Delta ABC \sim \Delta DEF$

$$\frac{AB}{AC} = \frac{DE}{DF} \quad \text{and} \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

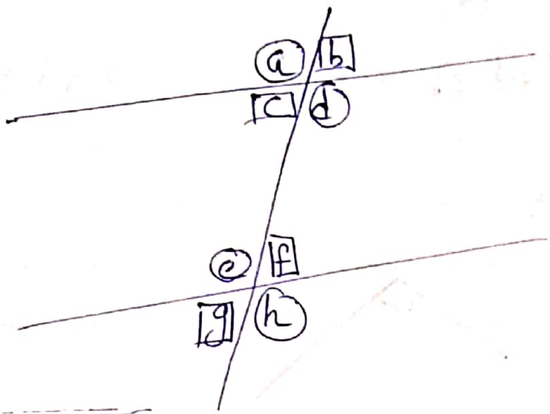
✓ If a Δ is scaled up in sides by a factor of k, its area gets scaled up by a factor of k^2.

GEOMETRY

> PARALLEL LINES

All big ∠'s are equal

$$\boxed{a = d = e = h}$$



$$\boxed{c + e = 180^\circ}$$

$$\boxed{d + f = 180^\circ}$$

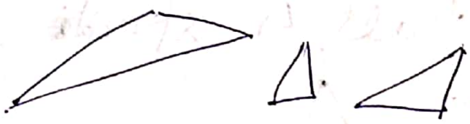
$$\boxed{b = c = f = g}$$

All small ∠'s are equal

$$\Downarrow$$

$$\boxed{\text{Any big} \angle + \text{Any small} \angle = 180^\circ}$$

> TRIANGLES



✓ The sum of any two sides is strictly greater than the third side.

$$\boxed{\text{Side 1} + \text{Side 2} > \text{Side 3}}$$

∴ If two sides of a triangle are known and we wish to know the range that the third side x would lie in,

$$\boxed{|\text{Side1} - \text{Side2}| < x < |\text{Side2} - \text{Side1}|}$$

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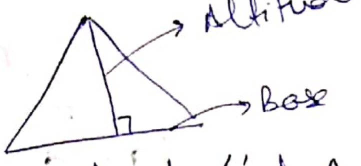
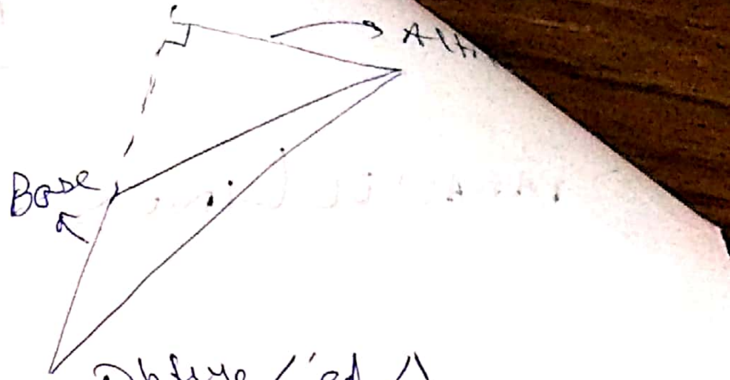
✓ Altitude :-

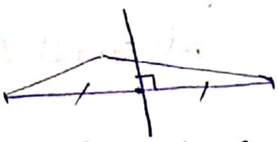


Acute ∠'ed Δ

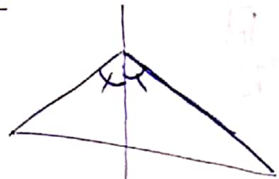


Obtuse ∠'ed Δ

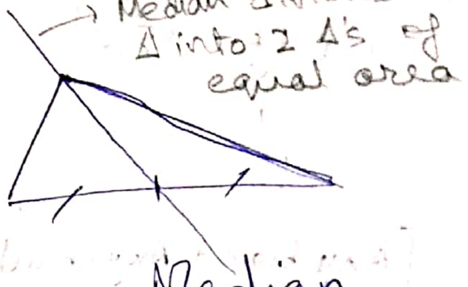
✓ Special Lines



Perpendicular Bisector



Angle Bisector

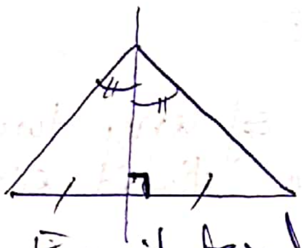


Median

✓ Only in isoceles & equilateral triangles are all 4 special lines the same

*: If it is known that a line plays more than one role, then for sure it is an isoceles Δ.

Isoceles



Equilateral

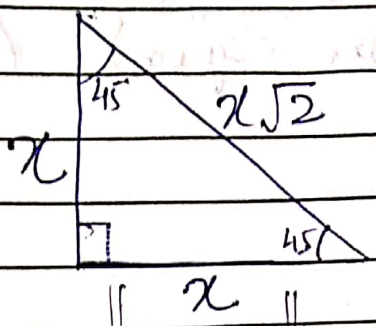
✓ An equilateral Δ is an isoceles Δ.

✓ If the Pythagorean Theorem applies to the sides of a Δ, then we know it is a right Δ.

✓ For sides with large magnitudes on right Δ's, scale down by taking the GCF, apply the Pythagorean Theorem, and then scale up by multiplying by the GCF

GEOMETRY

45-45-90 TRIANGLE (ISOSCELES RIGHT TRIANGLE)

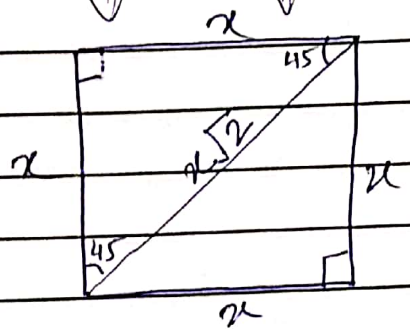


$45^\circ : 45^\circ : 90^\circ$

Leg : Leg : Hypotenuse

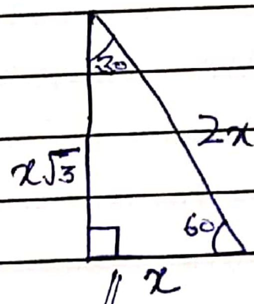
$1 : 1 : \sqrt{2}$

$x : x : x\sqrt{2}$



← Square

30-60-90 TRIANGLE & EQUILATERAL TRIANGLES

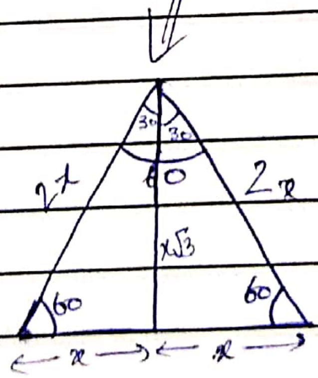


$30^\circ : 60^\circ : 90^\circ$

Short leg : Long leg : Hypotenuse

$1 : \sqrt{3} : 2$

$x : x\sqrt{3} : 2x$



← Equilateral Triangle

→ REGULAR POLYGON WITH 'n' SIDES

✓ All sides are equal (equilateral)

✓ All angles are equal (equiangular)

Pythagorean Triplets

✓ (3,4,5) → $\overset{\times 2}{(6,8,10)}$, $\overset{\times 3}{(9,12,15)}$, $\overset{\times 4}{(12,16,20)}$

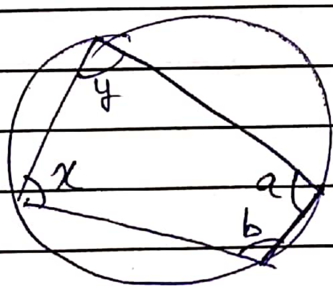
✓ (5,12,13) → $\overset{\times 2}{(10,24,26)}$

✓ (7,24,25)

✓ (8,15,17)

✓ Multiply the four by any positive integer to get another set.

Interior Angles Of A Quad. Inscribed In A Circle



$$\boxed{\begin{aligned} x+a &= 180^\circ \\ y+b &= 180^\circ \end{aligned}}$$

✓ Opposite angles sum up to 180°

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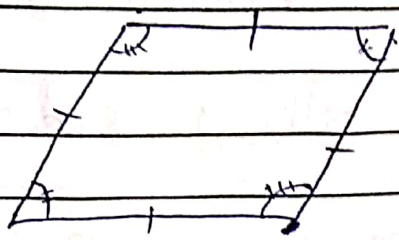
→ RHOMBUS

$\boxed{\text{Area} = bh}$

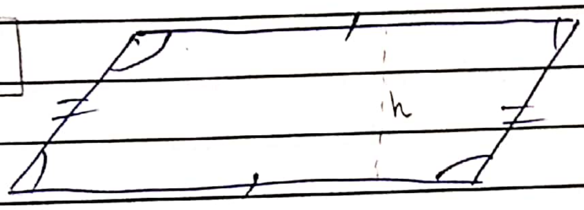
$\boxed{\text{Area} = \frac{d_1 d_2}{2}}$



- Opposite sides are equal
- Opposite angles are equal
- All sides are equal

→ PARALLELOGRAM

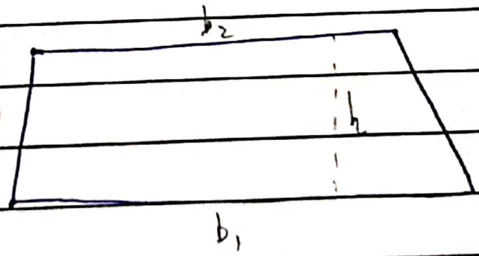
$\boxed{\text{Area} = bh}$



- Opposite sides are equal
- Opposite angles are equal.

→ TRAPEZOID / TRAPEZIUM

$\boxed{\text{Area} = \frac{(b_1 + b_2)h}{2}}$



- One pair of opposite sides is parallel

→ SUM OF INTERIOR ANGLES OF A POLYGON

n = no. of sides of the polygon

$$\boxed{\text{Sum of interior } \angle\text{'s} = (n-2) \times 180}$$

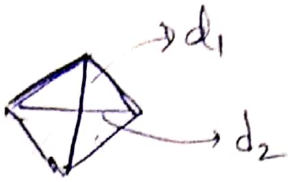
• NUMBER OF DIAGONALS OF A POLYGON

✓ In a polygon with 'n' sides,

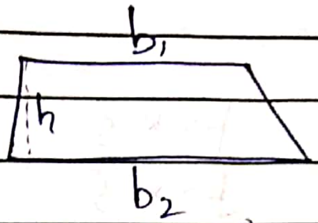
$$\boxed{\#\text{Diagonals} = \frac{n(n-3)}{2}}$$

AREA OF A RHOMBUS

$$\boxed{\text{Area} = \frac{d_1 \times d_2}{2}}$$



AREA OF A TRAPEZOID



$$\boxed{\text{Area} = \frac{(b_1 + b_2)}{2} \times h}$$

SURFACE AREA.

✓ $\boxed{\text{S.A. of a Cube} = 6s^2}$

✓ $\boxed{\text{S.A. of a Cuboid} = 2(lb + bh + hl)}$

MAXIMUM AREA & MINIMUM PERIMETER OF A POLYGON

✓ $\boxed{\text{Square}}$ → • Maximizes area for a given perimeter
AND
• Minimizes perimeter for a given area

✓ $\boxed{\text{A regular polygon with 'n' sides}}$ → • Maximizes area for a given perimeter
AND
• Minimizes perimeter for a given area.

⇒ MULTIPLE RATIOS

✓ C : A : L ⇒ C : A : L

3 : 2 : → ×5 15 : 10 :

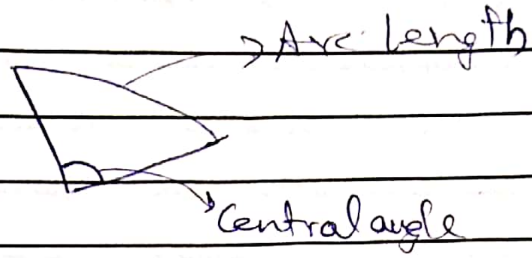
5 : : 4 → ×3 15 : : 12

15 : 10 : 12

⇒ C : A : L = 15n : 10n : 12n

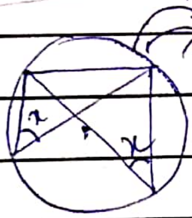
N

Circles & Sectors

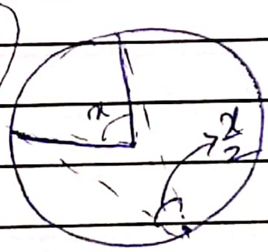


$$\boxed{\frac{\text{Central angle}}{360} = \frac{\text{Sector Area}}{\text{Circle Area}} = \frac{\text{Arc Length}}{\text{Circumference}}}$$

Inscribed Vs. Central Angle

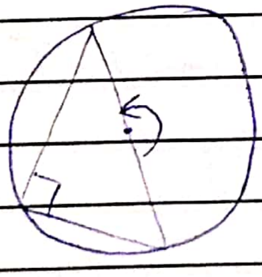


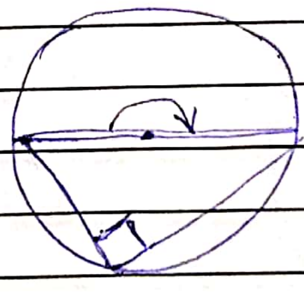
All angles inscribed by a chord (on the same side) are of equal measure

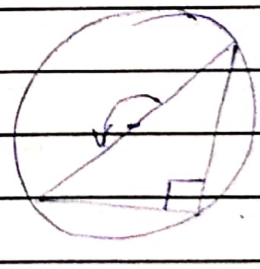


$$\boxed{\text{Inscribed } \angle = \frac{\text{Central} \angle}{2}}$$

Inscribed Triangle

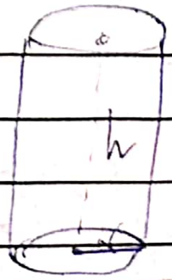






✓ If one of the sides of an inscribed Δ is the 'd' of the circle, then the Δ must be a right Δ.

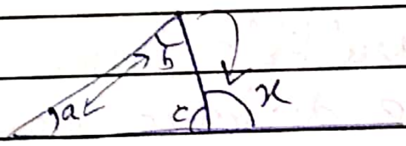
-> CYLINDERS



$$\boxed{S.A. = 2\pi r(h+r)}$$

$$\boxed{\text{Volume} = \pi r^2 h}$$

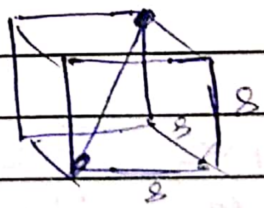
-> EXTERIOR ANGLES OF A TRIANGLE



$$\boxed{x = a+b}$$

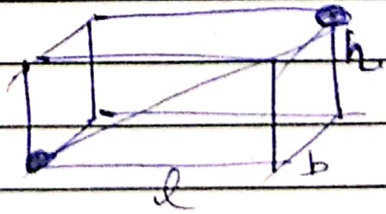
-> DIAGONAL OF A CUBE

$$\boxed{d = s\sqrt{3}}$$



-> DIAGONAL OF A CUBOID

$$\boxed{d = \sqrt{l^2+b^2+h^2}}$$



WORD PROBLEMS

RATE PROBLEMS

-> RELATIVE RATES -> If bodies leave at different times, then the one leaving first takes longer time.

Opp. Direction ✓

$\xrightarrow{r_1} \quad \xleftarrow{r_2}$

$$\boxed{r = r_1 + r_2}$$

$\xleftarrow{r_1} \quad \xrightarrow{r_2}$

But take care of 'approaching' and 'receding' gaps

Same Direction ✓

$\xrightarrow{r_1} \quad \xrightarrow{r_2}$

$$\boxed{r = |r_1 - r_2|}$$

✓ Set-up the DRT equation for the 'gap' between entities. This saves time.

-> AVERAGE RATE

✓ For avg. rate, calculate Total D (or W) and Total T separately, then,

$$\boxed{\text{Avg. } R = \frac{\text{Total D (or W)}}{\text{Total T}}}$$

✓ If rates for multiple trips for same (unknown) distance are given, take distance equal to LCM of rates.

R	T	D
r_1	$\left(\frac{LCM}{r_1}\right)$	$LCM(r_1, r_2)$
r_2	$\left(\frac{LCM}{r_2}\right)$	$LCM(r_1, r_2)$
	Total T	Total D

-> WORKING TOGETHER

✓ When multiple entities work together their rates get added ->

$$r_{a+b} = r_a + r_b$$

WORD PROBLEMS

→ CATEGORIES

- ✓ Age
- ✓ Work
- ✓ Growth & Decay
- ✓ Motion
  - Avg. Speed
  - Relative Motion
  - Multiple Traveller
- ✓ Sequences
  - Algebraic
  - A.P.
  - Recursive (Can't jump to n^{th} term)
- ✓ Sets
  - Venn Diagrams
    - 2-Set
    - 3-Set
  - Double Matrix
- ✓ Mixtures $\boxed{\dfrac{\text{Solute}}{\text{Soln.}} = \text{Concn.}}$

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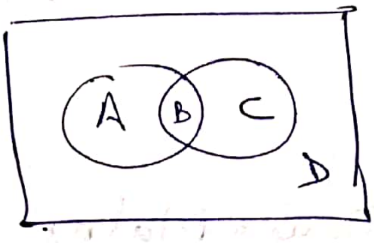
STRATEGIES

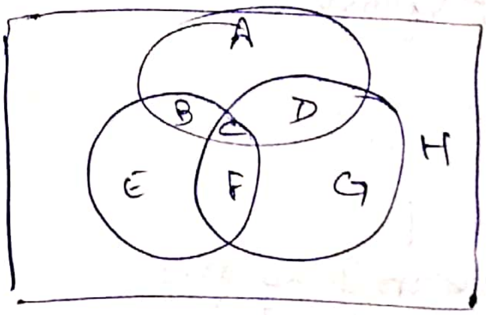
✓ If all choices are numerical, use backsolving

✓ If answer choices are in variables only (VIC's) and if your answer based on the algebra doesn't match any choice, try rearranging your answer to make it identical to one of the choices.

→ VENN DIAGRAMS

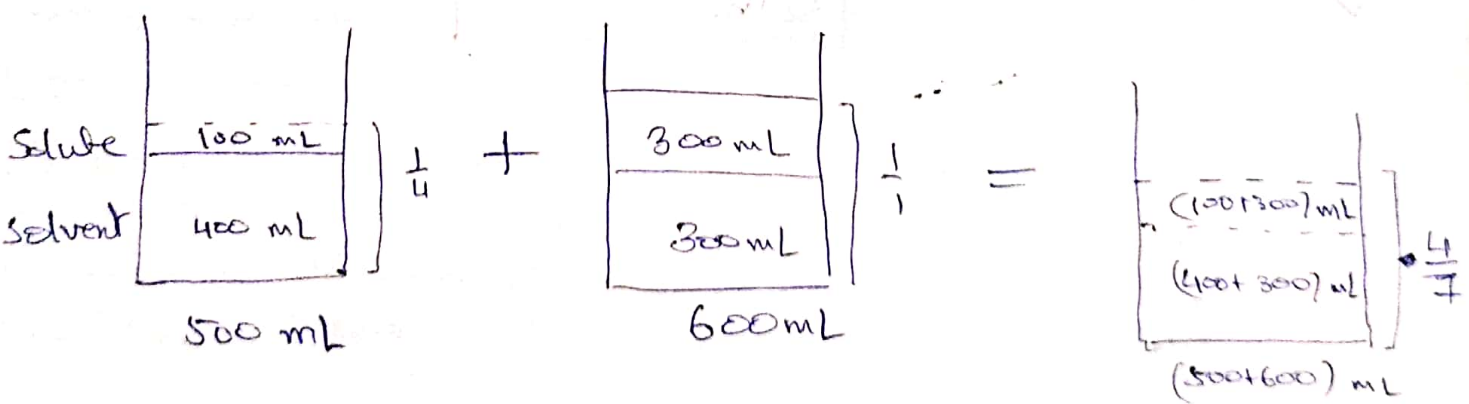
✓ Divide everything into smallest possible regions and then solve by naming each.





→ MIXTURES

✓ Draw diagrams of solutions to be mixed



unique elements } → set
Might have repeat elements } → list

STATISTICS

[An = S] → AnS Table

→ MEAN, MEDIAN, MODE (MEASURES OF CENTRE)

✓ $\boxed{\bar{X} = \frac{\Sigma x}{n}}$

✓ Md = Middlemost no. in an ordered list
→ Middle no. if n = odd
→ Mean of 2 middle nos. if n = even.

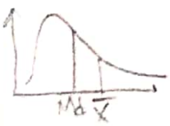
✓ Mode = Most frequent no. on the list.

✓ Every list has $\bar{X}$ & Md but not every list has a Mo. ∴ Mo is not that important.

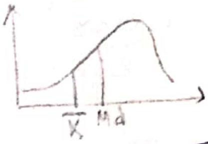
✓ Changing values of extremes changes $\bar{X}$ but doesn't change Md.

✓ For evenly spaced lists, $\bar{X}$ = Md
If a list is symetrical around the Md, then $\bar{X}$ = Md.

✓ For asymmetrical lists, Md sits still but the outliers pull the $\bar{X}$ towards them.
($\bar{X}$ gets pulled towards the 'tail' of the distn.)



• So, to compare $\bar{X}$ & Md in a given asymmetrical list, simply look at what direction the outliers lie in.



✓ For problems on $\bar{X}$, always think about sums (Σx).

✓ If all values multiplied by k, the mean becomes ($k\bar{X}$).

→ WEIGHTED AVERAGE

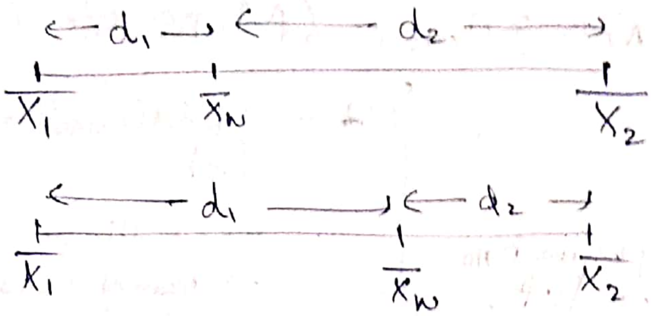
$$\boxed{\bar{X}_w = p_1\bar{X}_1 + p_2\bar{X}_2 + \ldots + p_n\bar{X}_n}$$

p_n → Proportion of group n
$\bar{X}_n$ → Mean of group n.

→ WEIGHTED AVERAGE FOR 2 GROUPS ONLY

$p_1 > p_2$

$p_1 < p_2$



$$\therefore \boxed{\frac{p_1}{p_2} = \frac{d_2}{d_1}}$$

→ RANGE & STANDARD DEVIATION (MEASURES OF SPREAD)

✓ $\boxed{\text{Range} = \text{Max} - \text{Min}}$ → Crude, tells very little

✓ Standard Deviation → Avg. spread of data around $\bar{X}$

- Subtract $\bar{X}$ from all values
- Square all resulting values
- Add them & divide by n.
- Take square root.

$$\boxed{\sigma = \sqrt{\frac{\Sigma(x_i - \bar{X})^2}{n}}}$$

(1, 2, 3, 4, 5)
(−2, −1, 0, 1, 2)
(4, 1, 0, 1, 4)
(10/5) = 2
$\sqrt{2}$

On an average, values are at a distance of $\sqrt{2}$ from $\bar{X} = 3$.

✓ Facts about σ

- Only (+ve) or zero, never (−ve)
- σ = zero only if all values are same.
- If all values are exactly the same dist. from the mean, this dist. is σ.
- A set with most values clustered towards the extremes will have a higher σ than one with most values equal to or close to $\bar{X}$.

- Adding or subtracting the same no. to every value doesn't affect σ.
- σ only deals with spacing b/w values, not where they are on the number line.

  If you imagine values as dots on the number line, they can be slided up or down or even reflected, as long as the spacing remains the same.

- If every value is multiplied by k, σ is also multiplied by k.

- Adding new values to the list changes σ accordingly
  - → If. ~~New Value~~ $|\bar{X} - NewValue| > \sigma$, then $\sigma \uparrow$
  - → If $|\bar{X} - NewValue| = \sigma$, σ is unchanged
  - → If $|\bar{X} - NewValue| < \sigma$, $\sigma \downarrow$.

If $\bar{X} = NewValue$ ⟸ → If $|\bar{X} - NewValue| = Zero$, $\sigma \downarrow$ the most.

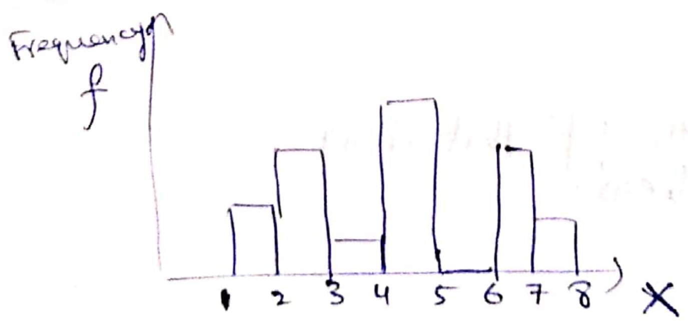
- σ can also be used as a unit of measurement (like percentiles).

  e.g. if mean score $\bar{X} = 50$, $X_1 = 60$, $\sigma = 2$ → then this score is 5σ above $\bar{X}$ and is pretty impressive

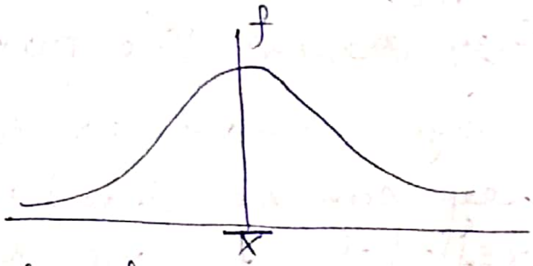
  but if $\bar{X} = 50$, $X_1 = 60$, $\sigma = 20$ } → this score is only $\left(\frac{\sigma}{2}\right)$ above $\bar{X}$ and is good but not impressive.

⇨ HISTOGRAM

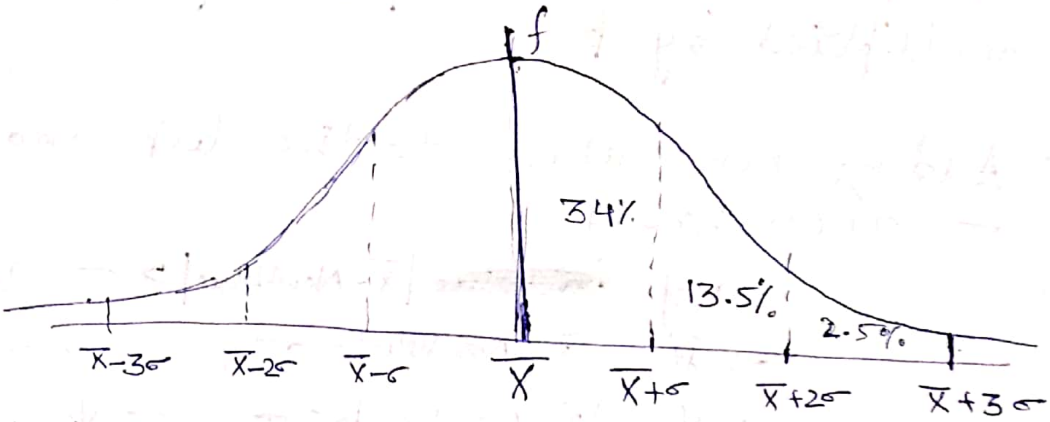
✓ Distribution at a glance.



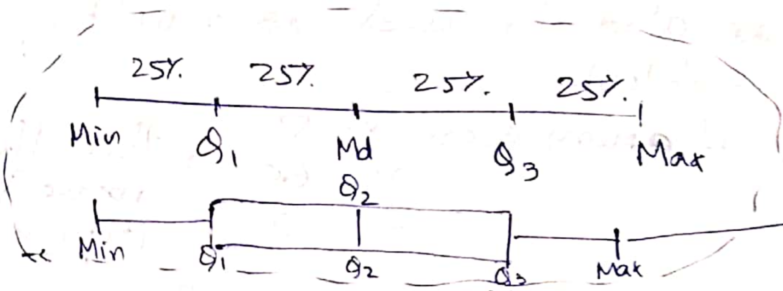
⇒ NORMAL DISTRIBUTION (BELL CURVE) (FOR POPULATIONS)



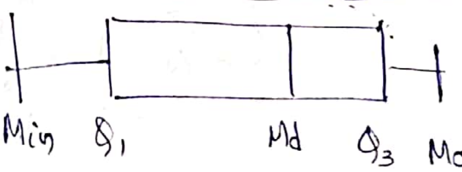
✓ All natural phenomena follow the bell curve
✓ Populations of mass-produced items follow it.



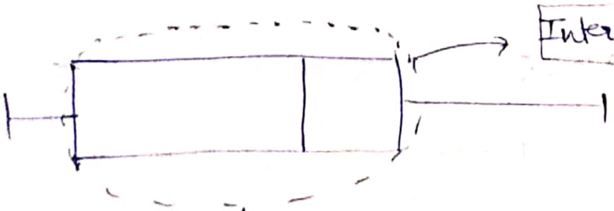
⇒ QUARTILES & BOXPLOTS (FOR POPULATIONS)



→ Boxplot



→ Asymmetric boxplot.
(Boxplots would most likely be asymmetric, but they always divide the popⁿ into 4 equal parts).



→ Interquartile Range $= Q_3 - Q_1$

↓
Middle 50% of the popⁿ that doesn't contain outliers.

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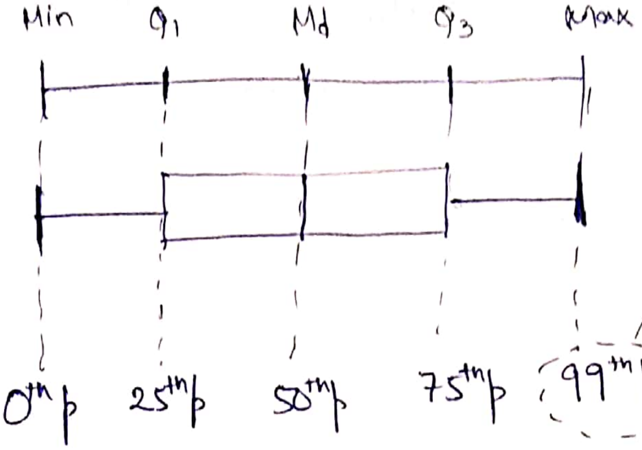
PERCENTILES

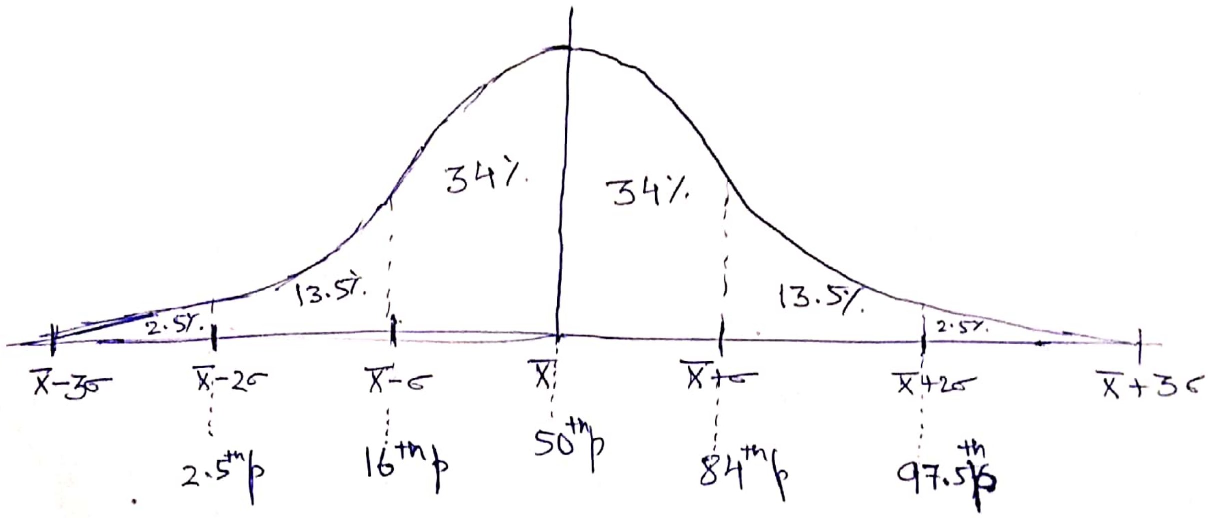
- If X is at the p^{th} percentile, then X is above p% of the ~~other~~ total values.

→ For Quartiles & Boxplots (Populations)



Can't have a 100^{th} p because ~~the~~ the X at 100^{th} p would be above 100% of the total values, which is impossible

→ For Normal Distributions (Populations)



DATA INTERPRETATION

> GENERAL

✓ 3-4 questions in every quant section.

✓ No quant. comparison. (only multiple choice, multiple answer, and numeric entry).

✓ All DI graphs are drawn to scale.

> STRATEGY

✓ Understand the story of the graph.

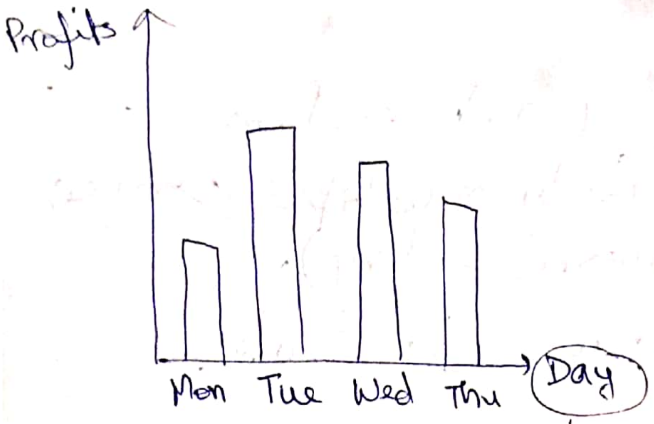
- Read accompanying text.
- Notice units of axes.
- Whether axes start at zero or have a kink.
- Whether axes increase by equally spaced intervals or some other intervals. (or if they are in descending order)
- Look for patterns
- Look for correlation and (visual) stochastic functional relationships, if any.

✓ Check units in the question.

✓ Check form of answer choices before calculating (e.g. fractions or percents).

✓ If choices are spread widely, feel free to estimate.

> Bar Graphs Vs Histograms



Discrete, dummy (qualitative) variable

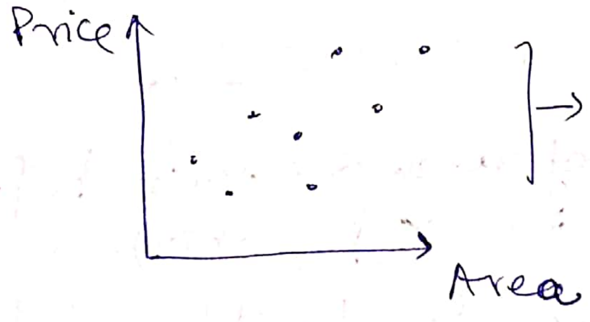
y-axis is always frequency

Continuous, numerical (quantitative) variable.

> Scatter Plots

✓ When each item in a popⁿ can be classified simultaneously by 2 different <u>numerical</u> variables, ~~they~~ it can be represented on a scatter plot

- e.g. Each house < Price, Area



→ Each dot is a house

✓ Correlation in the whole set <u>does not mean</u> that the pattern will be obeyed between every possible pair of points in the set.

- It is easy to spot pairs of elements that don't obey it.